



# Hardware Synthesis with Petri Nets: Appendix

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# Appendix

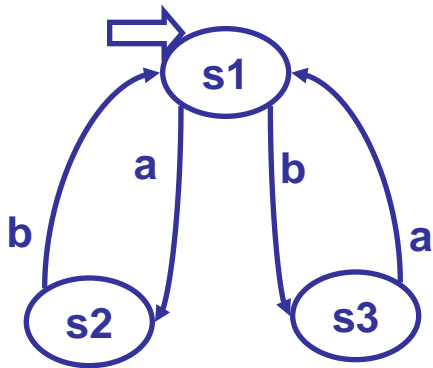
- **Synthesis of LPNs from Transition Systems (Using Region Theory)**
- **Direct synthesis of circuits from LPNs**
- **Examples**
- **Tools**

# Synthesis from transition systems

- Modelling behaviour in terms of a sequential capture – *Transition System*
- Synthesis of LPN (distributed and concurrent object) from TS (using *theory of regions*)
- Examples: one place buffer, counterflow pp

# Transition Systems

Original TS specification



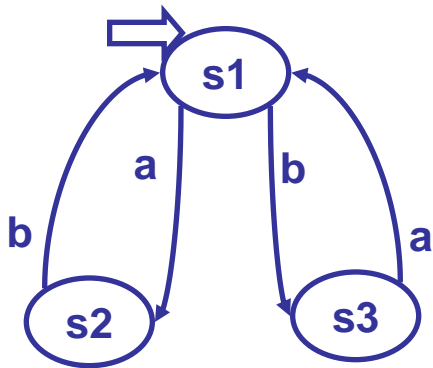
Not (semi-)elementary

The relationship between Transition Systems and Petri nets and conditions for synthesizability of a PN from a TS are based on *Theory of Regions*

(Ehrenfeucht, Rozenberg, Nielsen, Thiagarajan, Reisig, Mukund, Darondeau et al.)

# Transition Systems and regions

Original TS specification

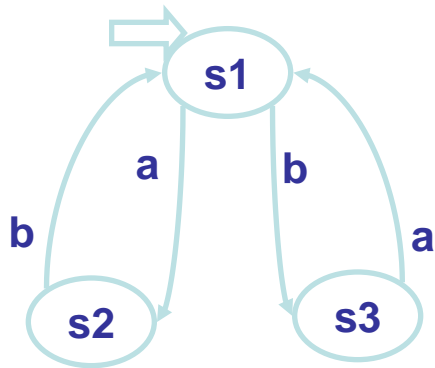


Not (semi-)elementary

No non-trivial  
regions!

# Transition Systems and regions

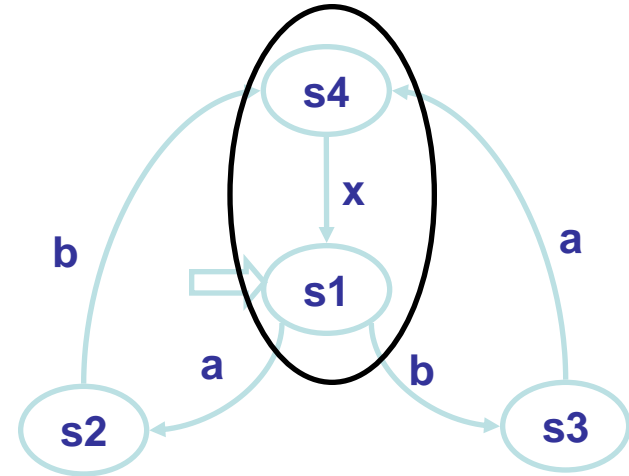
Original TS specification



Splitting  
states

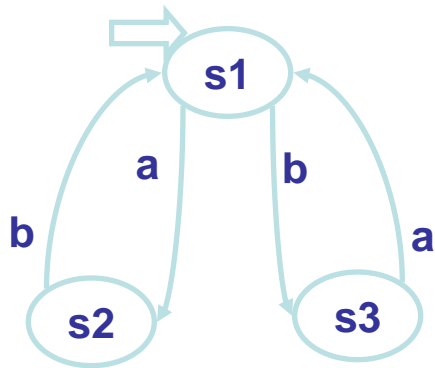


Inserting  
dummy  
events:  
(x)



# Transition Systems and regions

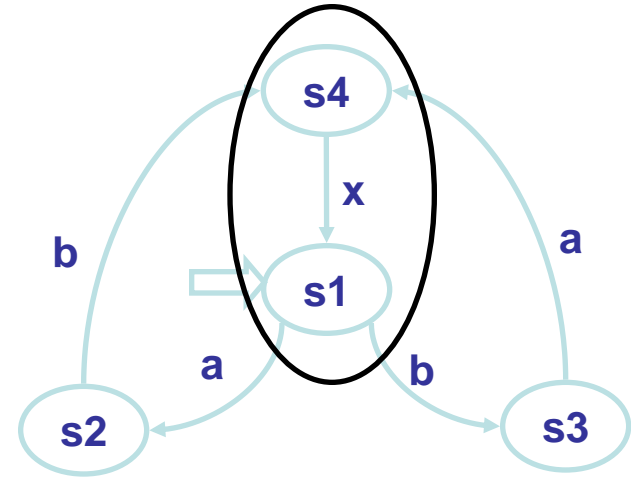
Original TS specification



Splitting  
states



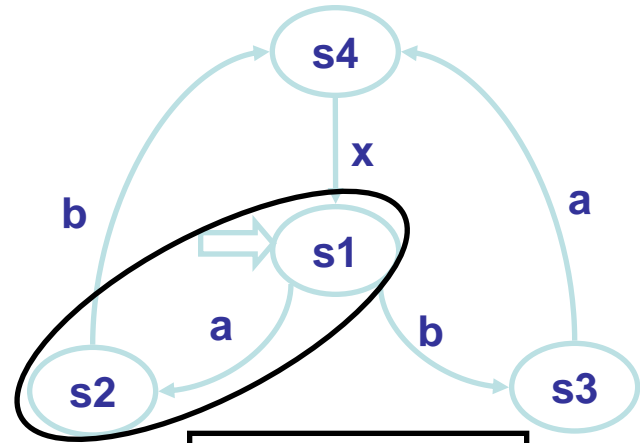
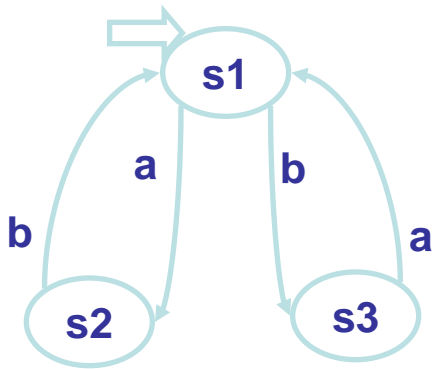
Inserting  
dummy  
events:  
(x)



This transformation preserves observational equivalence

# Transition Systems and regions

Original TS specification

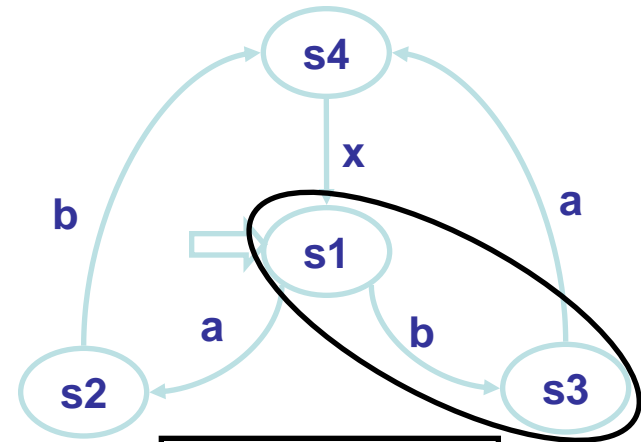
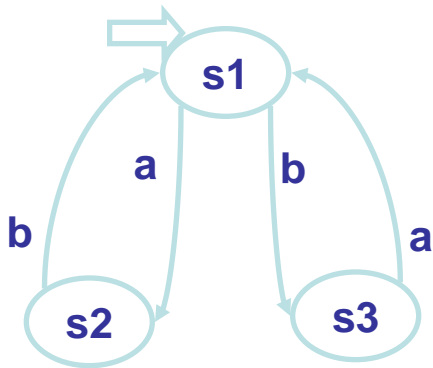


**Region r1:**  
exit(b)  
enter(x)  
no-cross(a)



# Transition Systems and regions

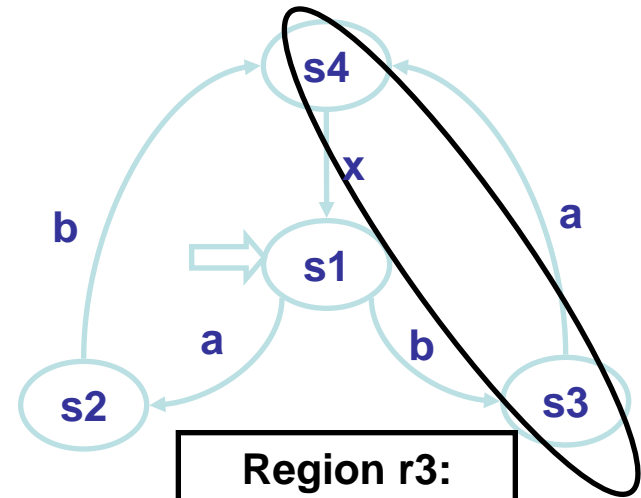
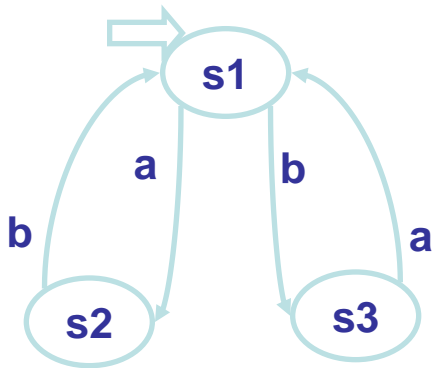
Original TS specification



**Region r2:**  
exit(a)  
enter(x)  
no-cross(b)

# Transition Systems and regions

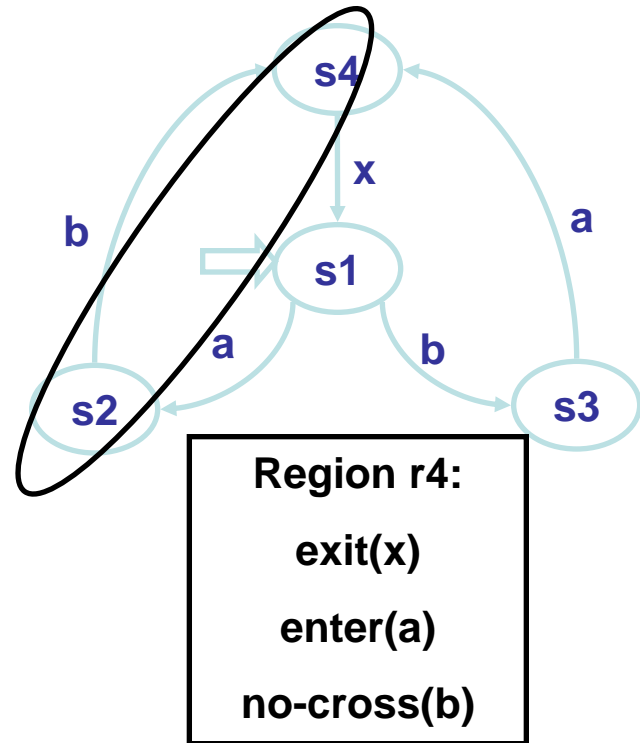
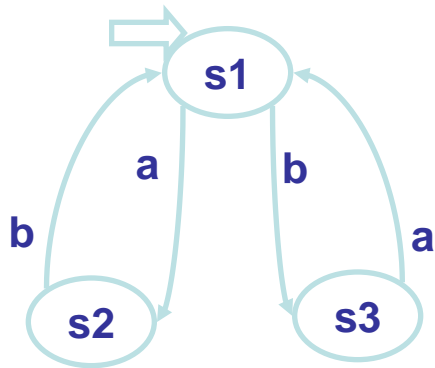
Original TS specification



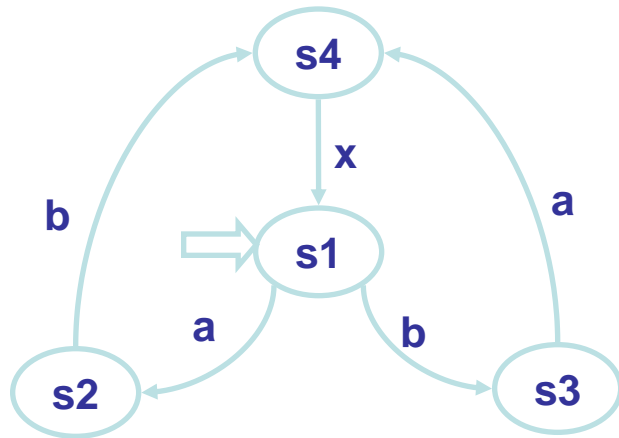
**Region r3:**  
exit(x)  
enter(b)  
no-cross(a)

# Transition Systems and regions

Original TS specification



# From Transition System to LPN



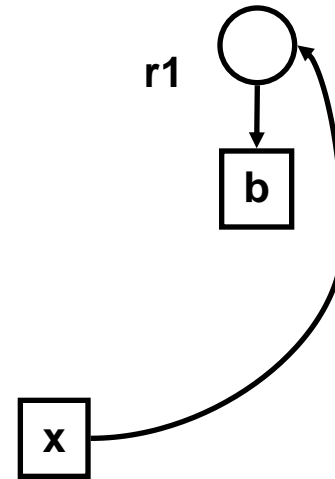
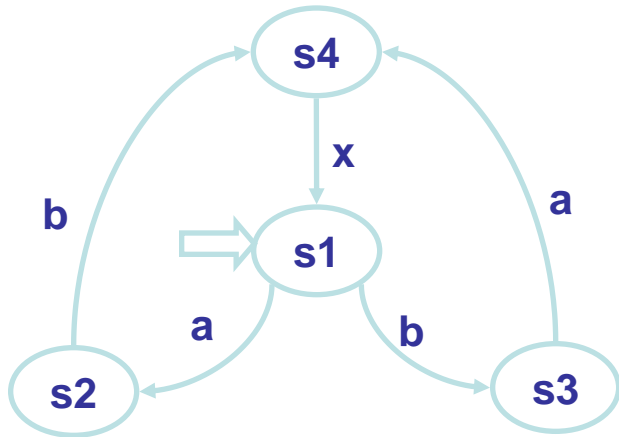
**Regions in the TS are associated with places in the LPN**

**Events are associated with transitions**

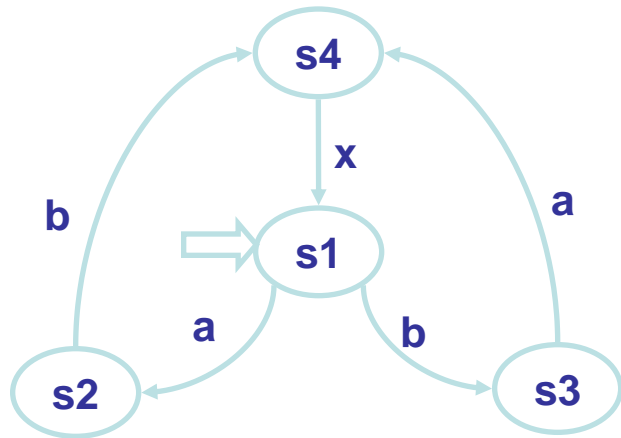
**Exit/entry/no-cross relations are associated with pre/post relations**

# From Transition System to LPN

r1: exit (b), enter(x), no-cross(a)

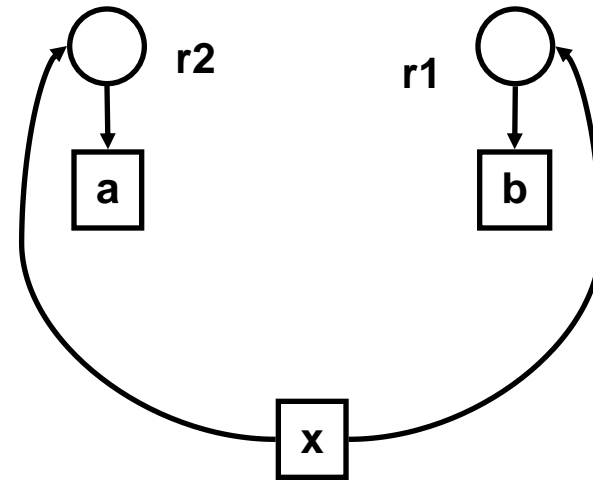


# From Transition System to LPN

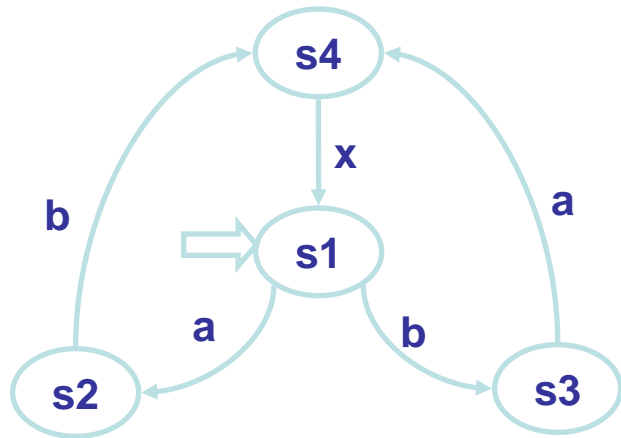


r1: exit (b), enter(x), no-cross(a)

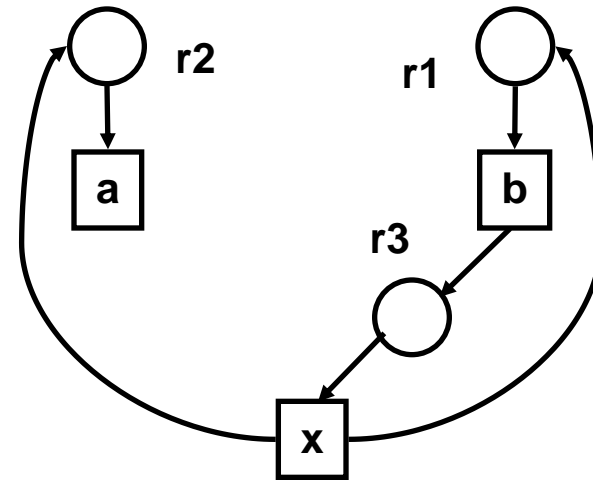
r2: exit (a), enter(x), no-cross(b)



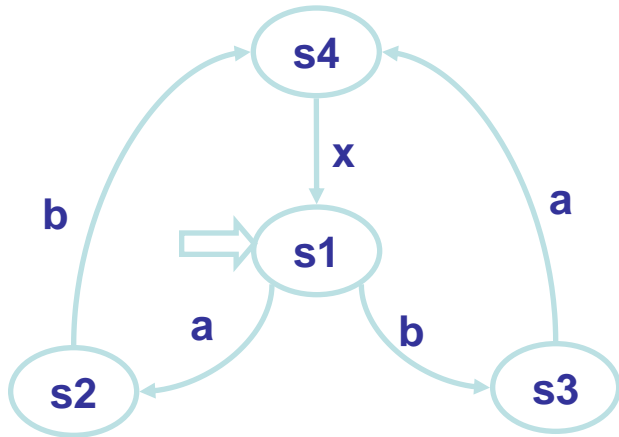
# From Transition System to LPN



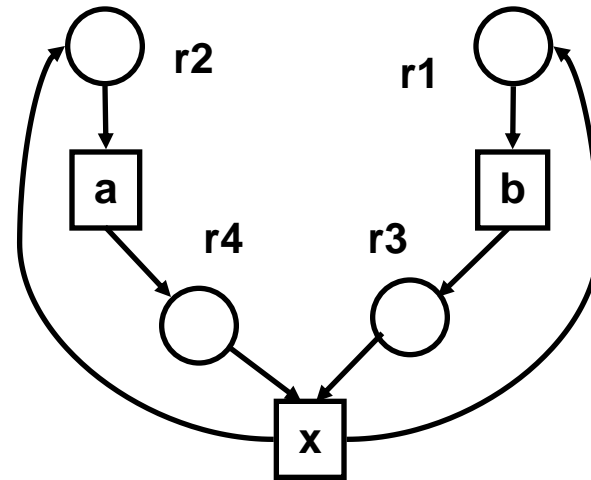
- r1: exit (b), enter(x), no-cross(a)
- r2: exit (a), enter(x), no-cross(b)
- r3: exit (x), enter(b), no-cross(a)



# From Transition System to LPN



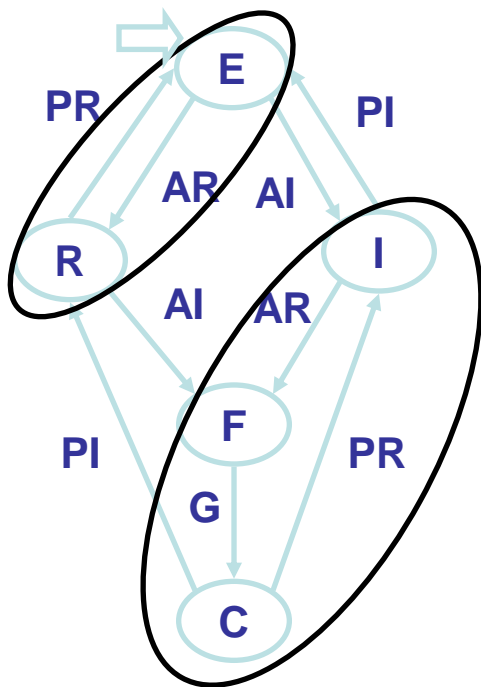
- r1: exit (b), enter(x), no-cross(a)
- r2: exit (a), enter(x), no-cross(b)
- r3: exit (x), enter(b), no-cross(a)
- r4: exit (x), enter(a), no-cross(b)





# Example: counterflow pipeline

Molnar's 5-state TS:



Examples of regions:

$r1 = \{E, R\}$  – pre-region(AI), post-region(PI)

$r2 = \{I, F, C\}$  – pre-region(PI), post-region(AI), co-region(G)

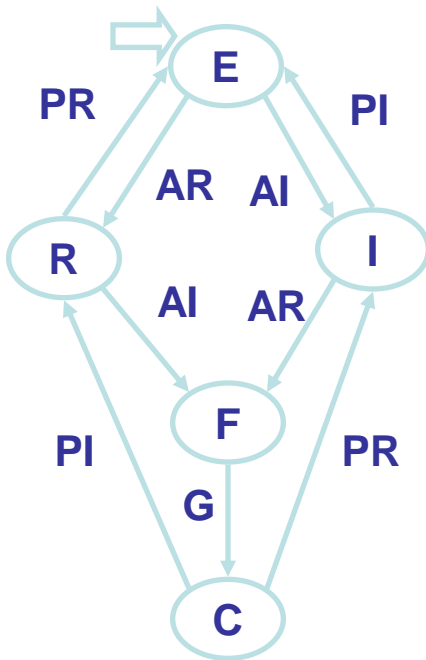
Notation: exit(a) -> pre-region(a), entry(a) -> post-region(a), inside(a) -> co-region(a)

Violation of semi-elementarity:

1. Intersection of pre-regions (only  $r2!$ ) for PI  $\{I, F, C\}$  is not equal to Excitation Region for PI  $\{I, C\}$
2. Intersection of pre-regions (empty!) for G is not equal to Excitation Region for G  $\{F\}$

# Example: counterflow pipeline

Molnar's 5-state TS:

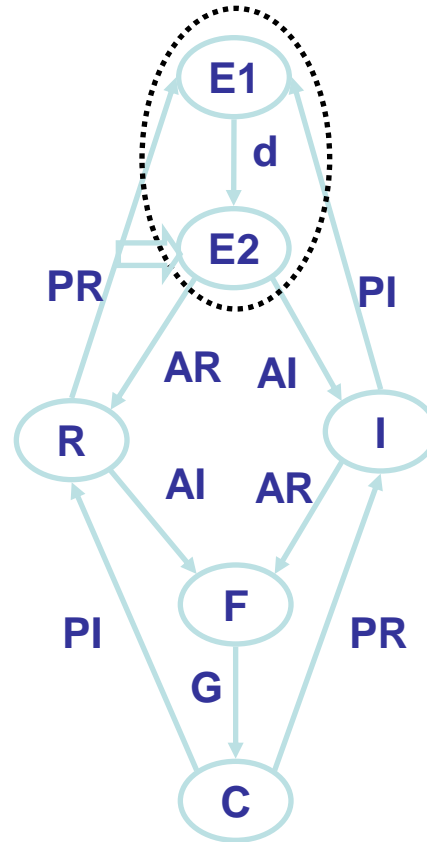
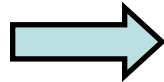
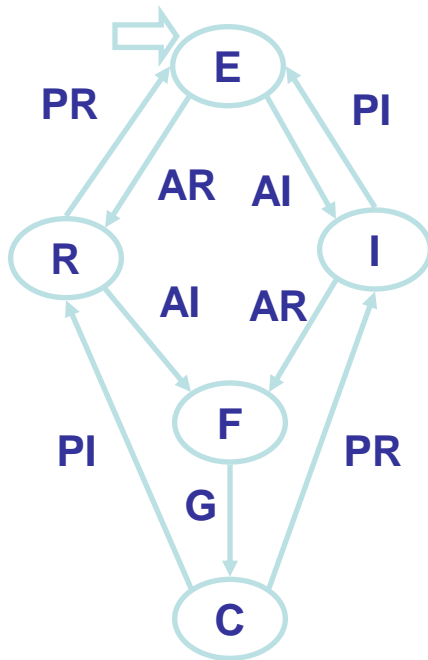


Solution:

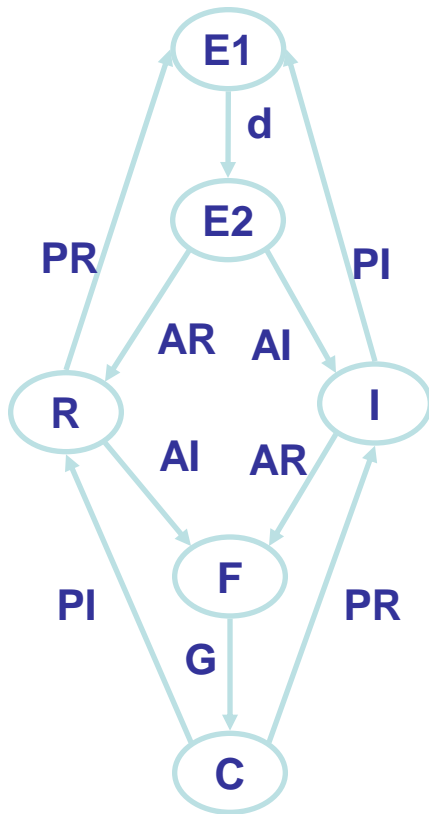
Split a state (E) and insert a silent action (d), preserving behavioural (observational) equivalence

# Example: counterflow pipeline

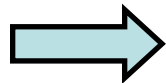
Molnar's 5-state TS:



# Example: counterflow pipeline



Semi-elementary TS



Minimal set of regions:

$$r1 = \{E1, E2, I\} \leftarrow \text{pre}(AR), \text{post}(PR)$$

$$r2 = \{E1, E2, R\} \leftarrow \text{pre}(AI), \text{post}(PI), \text{co}(d)$$

$$r3 = \{R, F, C\} \leftarrow \text{pre}(PR), \text{post}(AR), \text{co}(G)$$

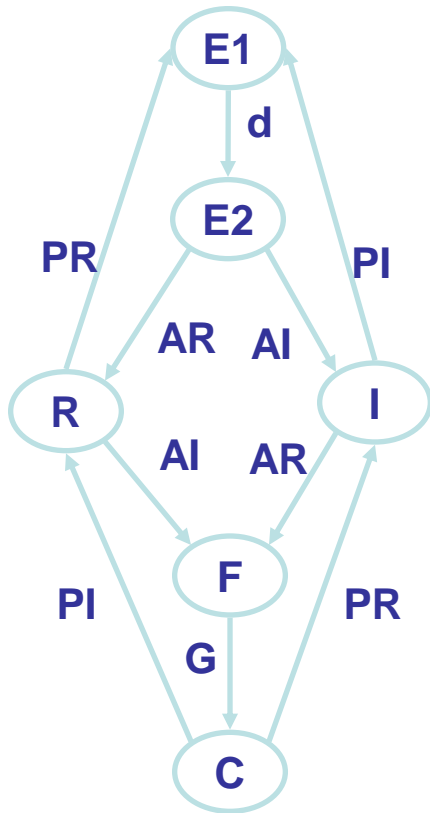
$$r4 = \{I, F, C\} \leftarrow \text{pre}(PI), \text{post}(AI)$$

$$r5 = \{E2, I, C\} \leftarrow \text{pre}(PI, AR), \text{post}(G, d)$$

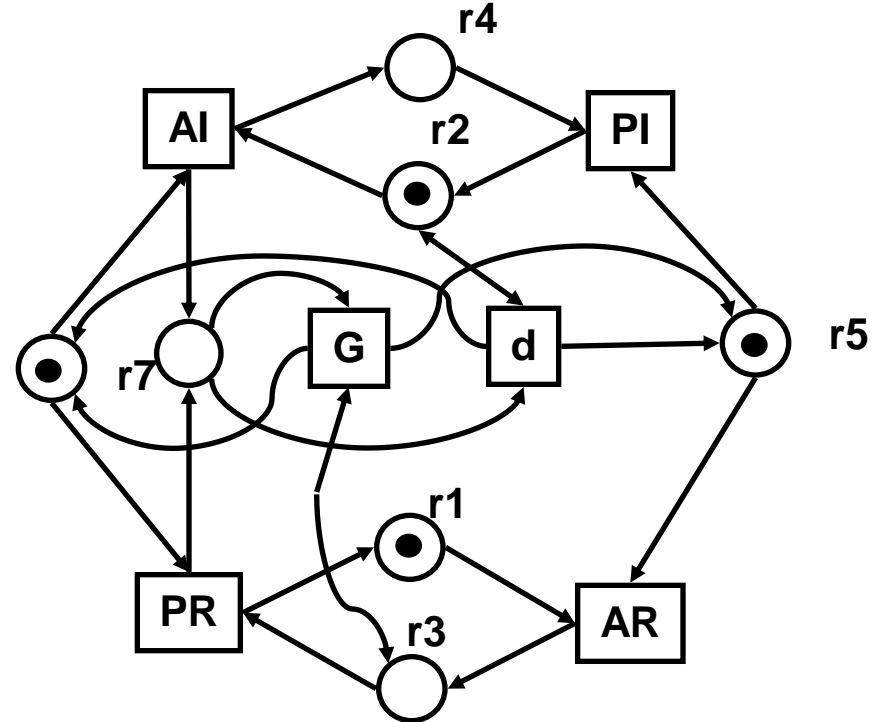
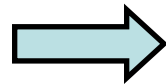
$$r6 = \{E2, R, C\} \leftarrow \text{pre}(PR, AI), \text{post}(G, d)$$

$$r7 = \{E1, I, F\} \leftarrow \text{pre}(G, d), \text{post}(PR, AI)$$

# Example: counterflow pipeline



Semi-elementary TS



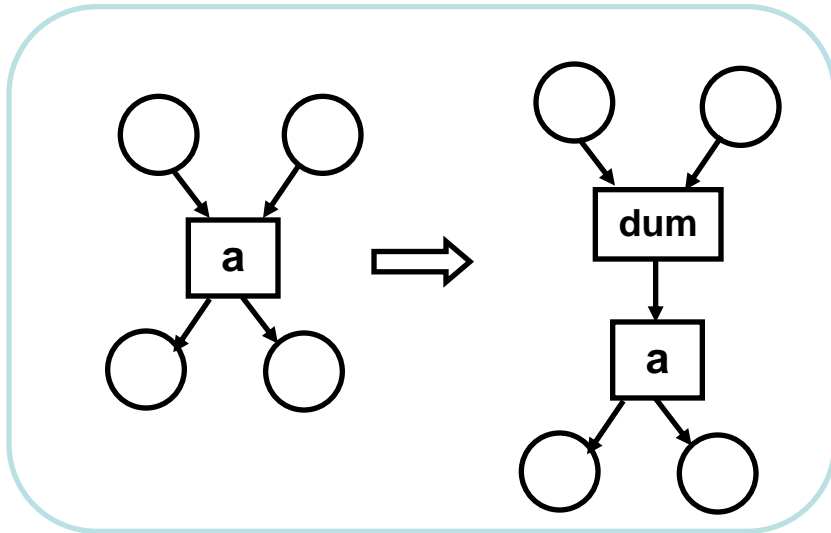
Semi-elementary Petri net

# Refinement at the LPN level

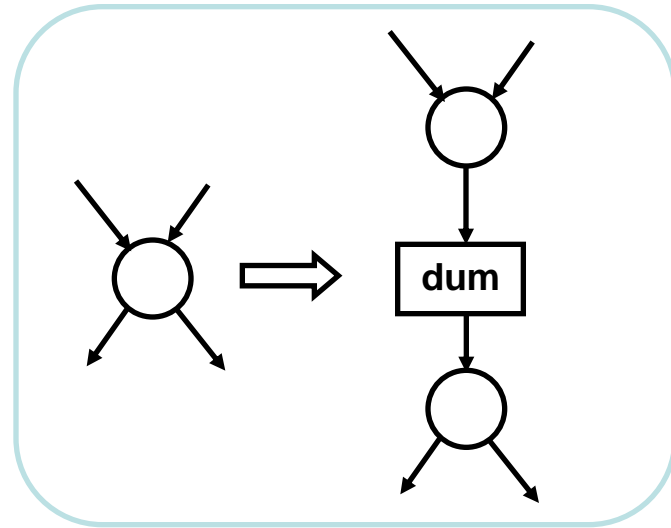
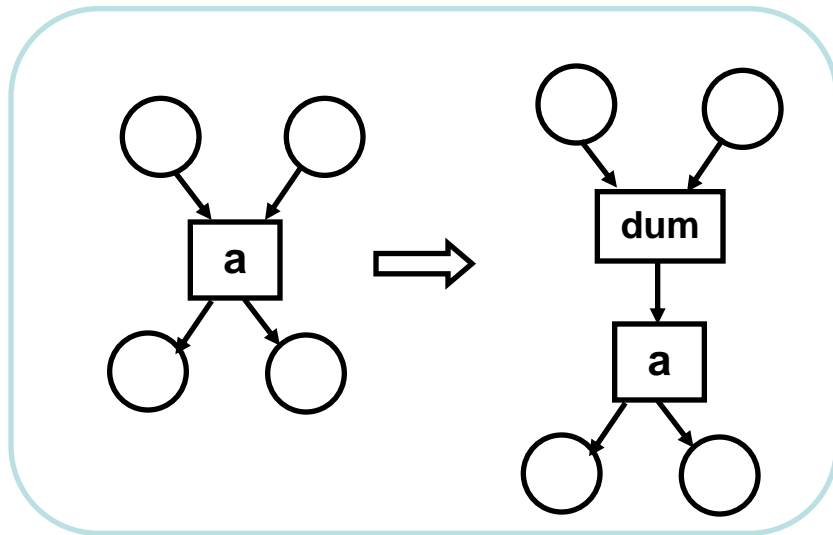
- Examples of refinements:
  - Introduction of “silent” events
  - Handshake refinement
  - Signalling protocol refinement (return-to-zero versus non-return-to-zero)
  - Arbitration refinement (Appendix)

*All these refinements must preserve **behavioural equivalence** (discussed below) and some other properties at the STG level (discussed later)*

# Structural refinement in LPN

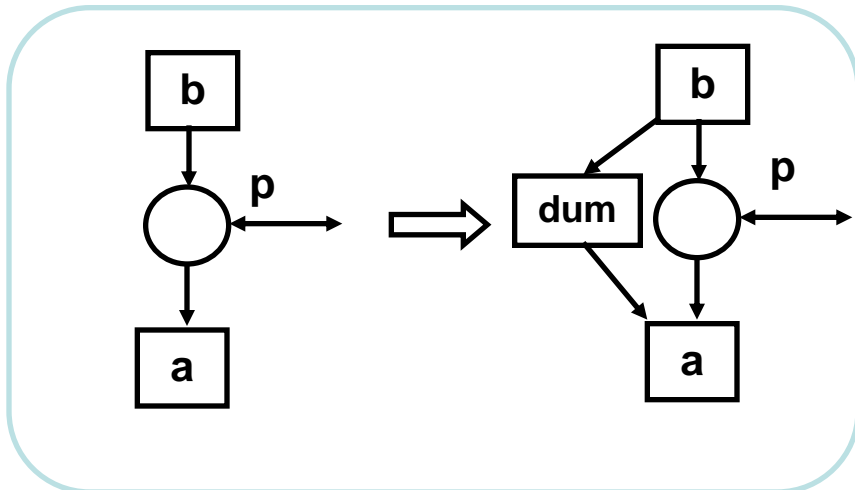
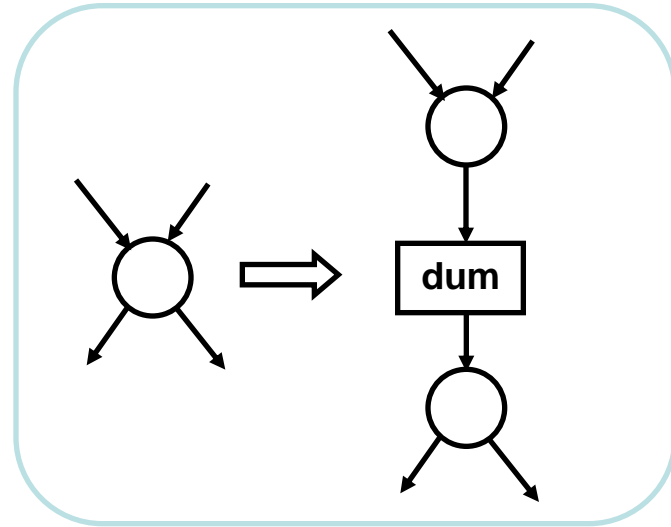
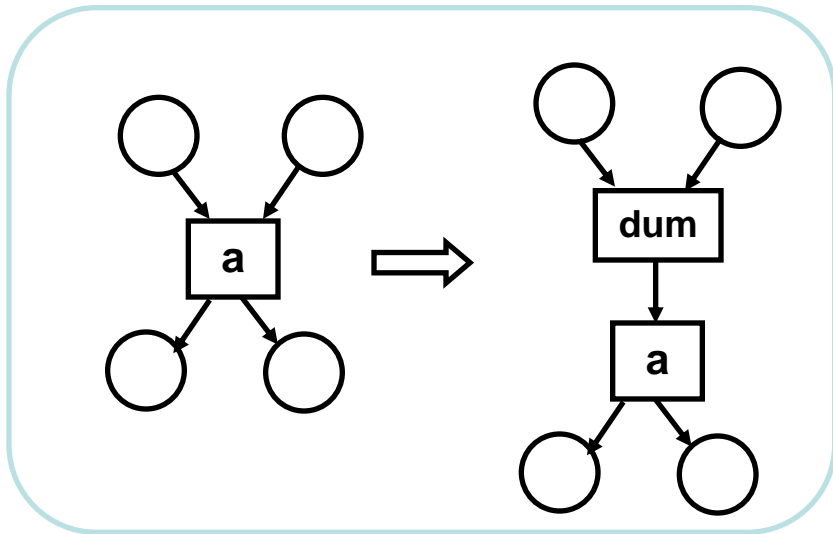


# Structural refinement in LPN

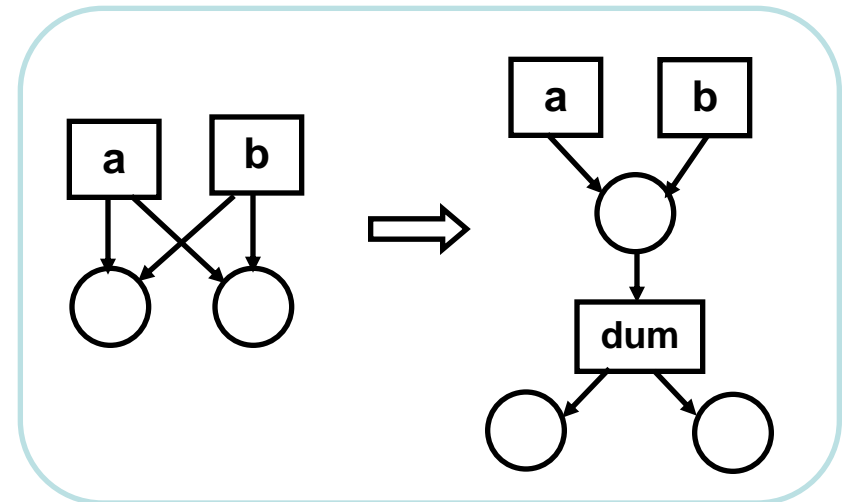
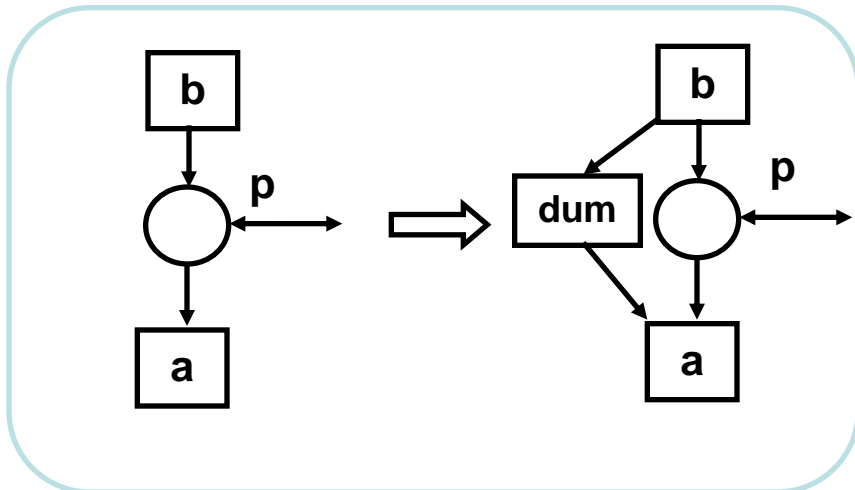
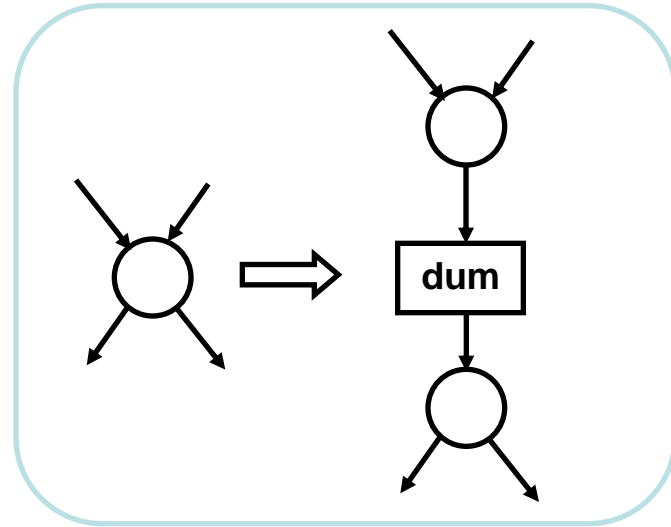
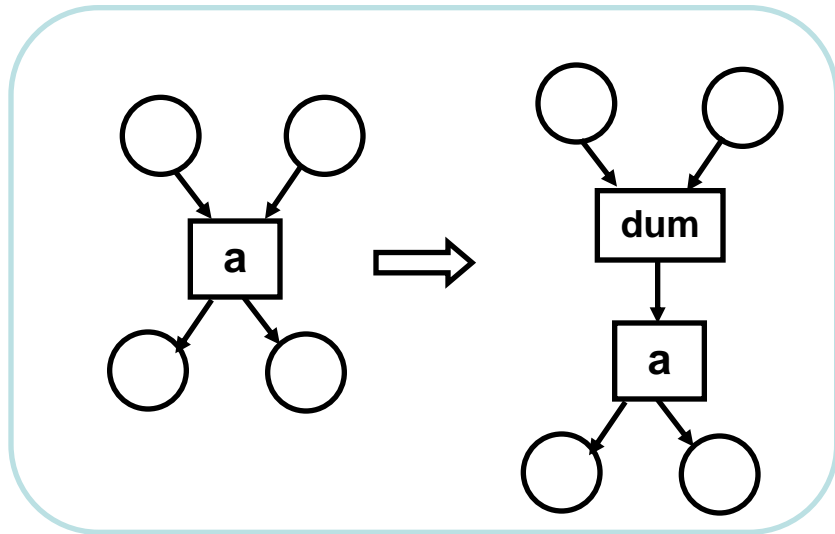




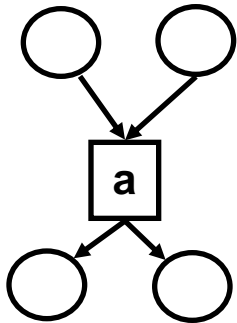
# Structural refinement in LPN



# Structural refinement in LPN

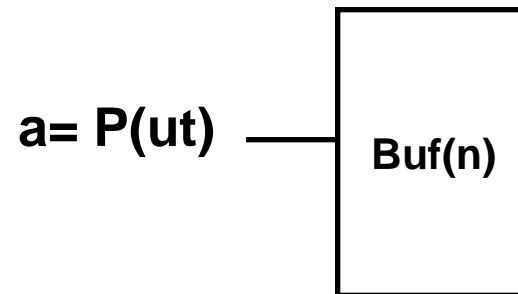


# Handshake refinement



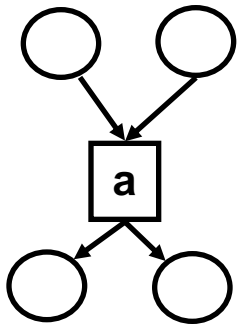
Let abstract event (action) “a” be associated with some port of the control circuit

E.g.

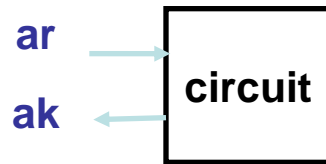


This may lead to the following refinements at the circuit level

# Handshake refinement

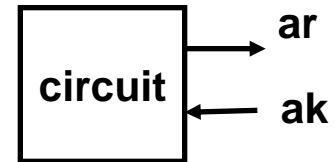


**Passive handshake**

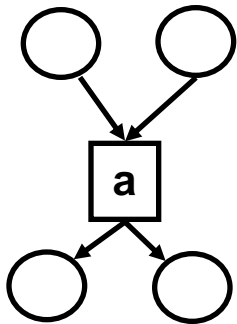


**Environment  
produces (first)  
request**

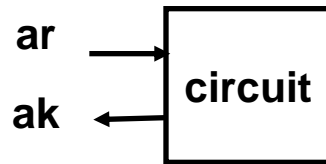
**Active handshake**



# Handshake refinement

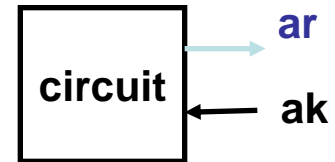


Passive handshake

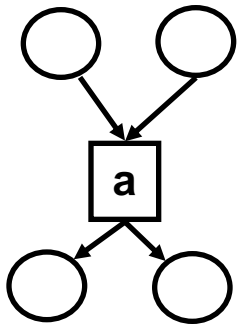


Environment  
produces (first)  
request

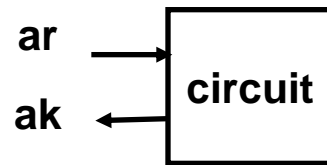
Active handshake



# Handshake refinement

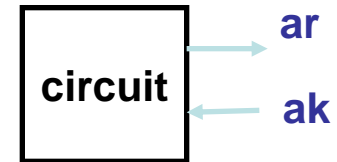


**Passive handshake**



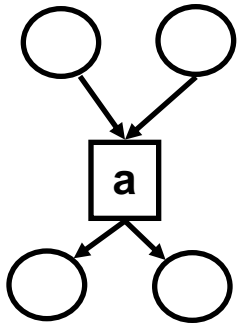
**Environment  
produces (first)  
request**

**Active handshake**

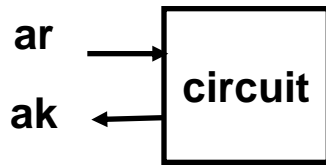


**Circuit produces  
(first) request**

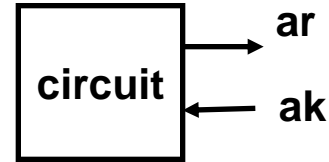
# Handshake refinement



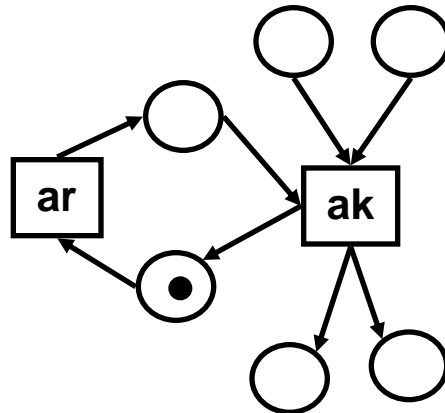
Passive handshake



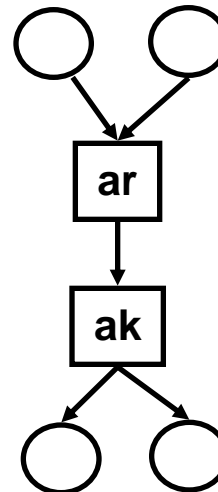
Active handshake



Environment produces (first) request

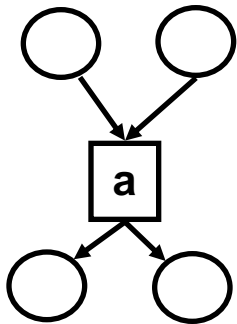


Circuit produces (first) request

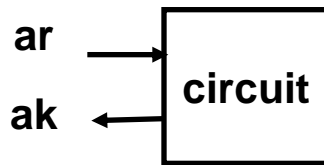


Two phase, non-return-to-zero (NRZ) protocol

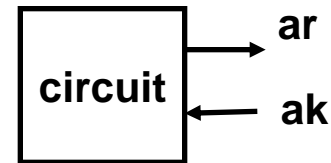
# Handshake refinement



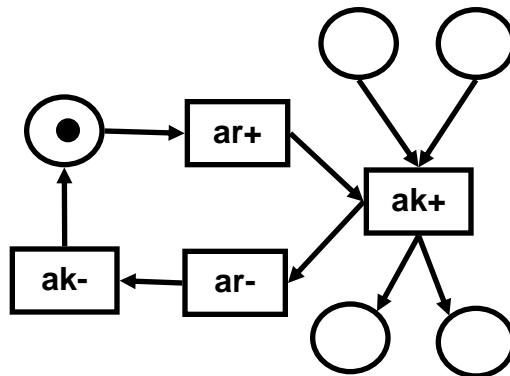
Passive handshake



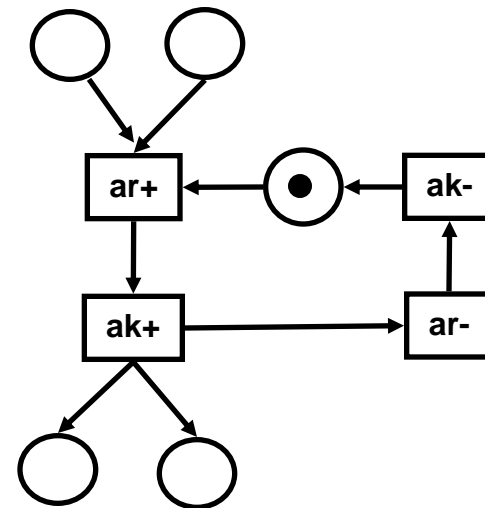
Active handshake



Environment produces (first) request



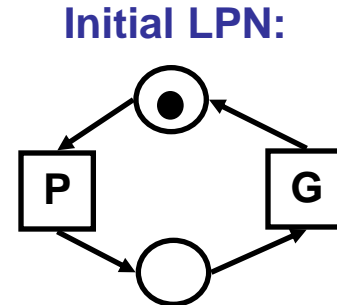
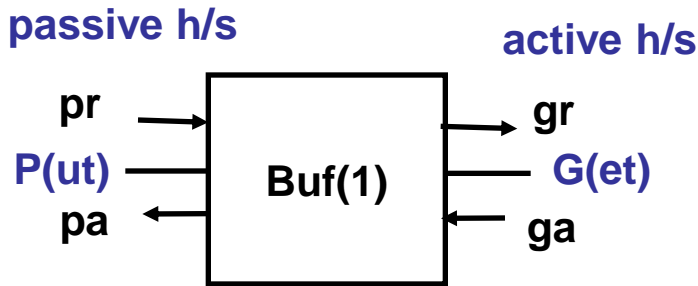
Circuit produces (first) request



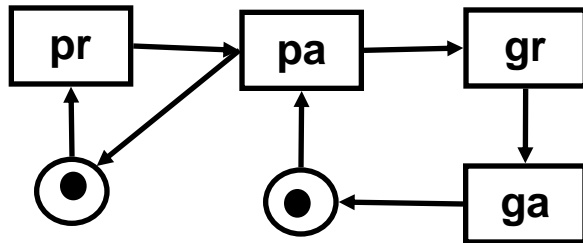
Four phase,  
return-to-zero  
(RTZ) protocol



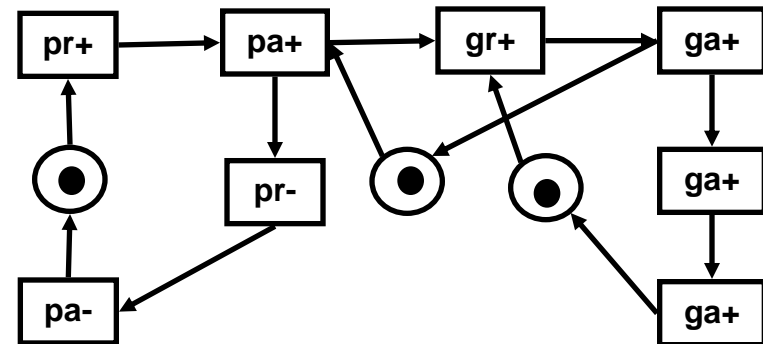
# Handshake refinement example



Two phase (NRZ) protocol:



Four phase (RTZ) protocol:

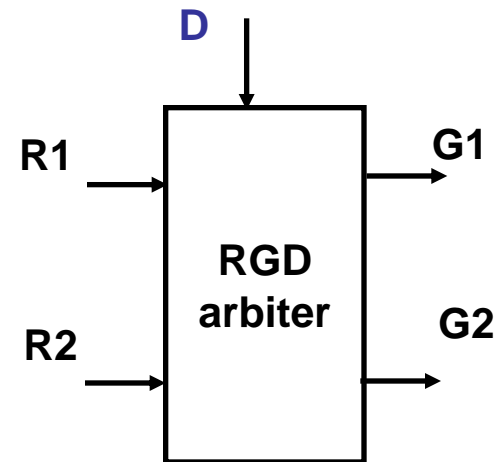


Subsequent transformations are possible at STG level –  
 e.g. re-shuffling of non-critical (resetting) transitions  
 (discussed later)

# Arbitration refinement

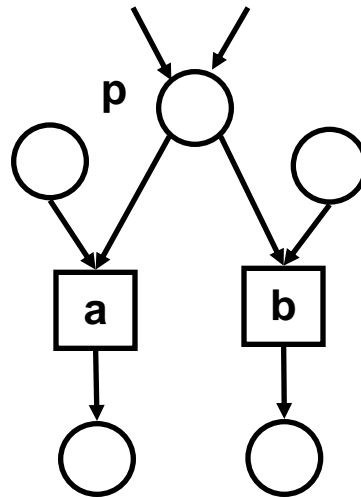
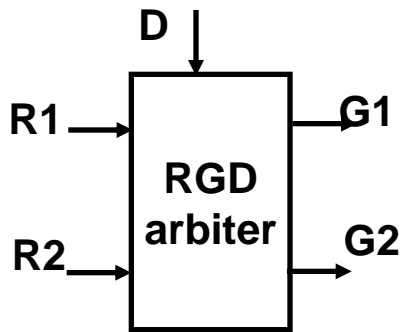
- Asynchronous circuits often require elements to resolve *conflicts* which are intentionally “pre-programmed” in specifications
- These elements are *similar to semaphores* (etc.) in concurrent programs
- These elements are *different from logical gates* because they involve *internally analogue* components
- The LPN model must be refined to explicitly “*factorise*” *non-persistent behavior* from the rest of the model – the latter can be synthesized using logic gates

E.g. Request-Grant-Done (RGD) arbiter



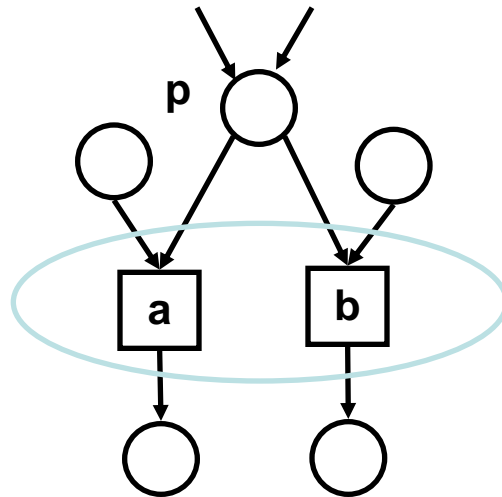
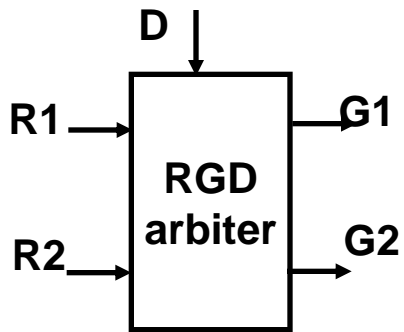
# Arbitration refinement

E.g. Request-Grant-Done (RGD) arbiter



# Arbitration refinement

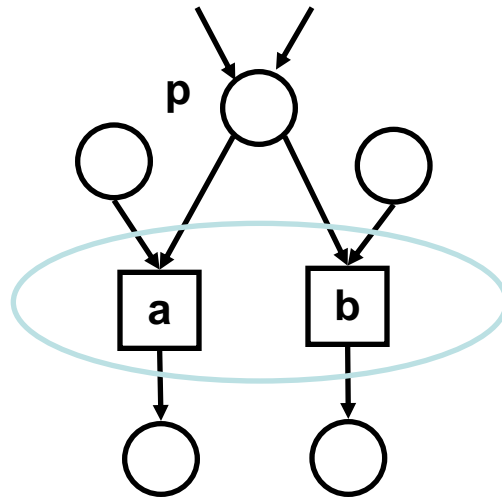
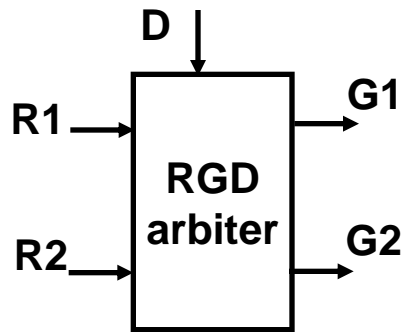
E.g. Request-Grant-  
Done (RGD) arbiter



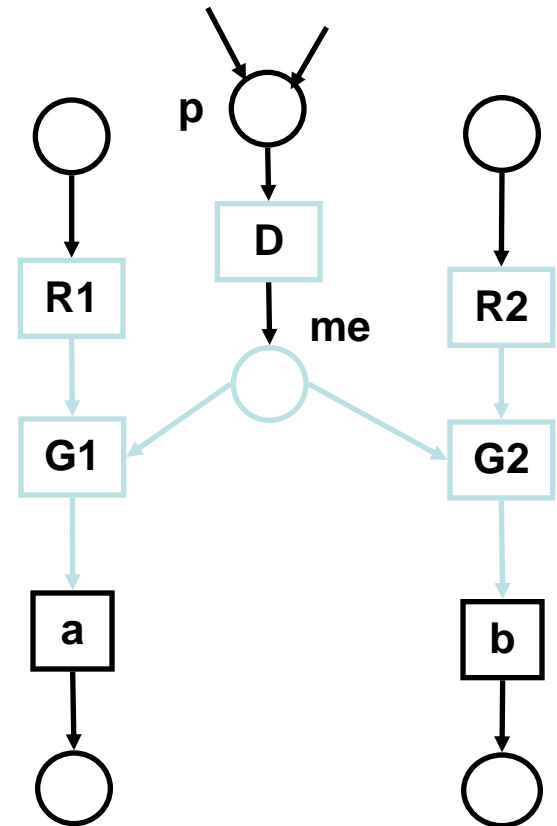
Assume **a** and **b** are circuit  
actions that are in conflict  
(may disable each other) and  
need to be protected

# Arbitration refinement

E.g. Request-Grant-  
Done (RGD) arbiter

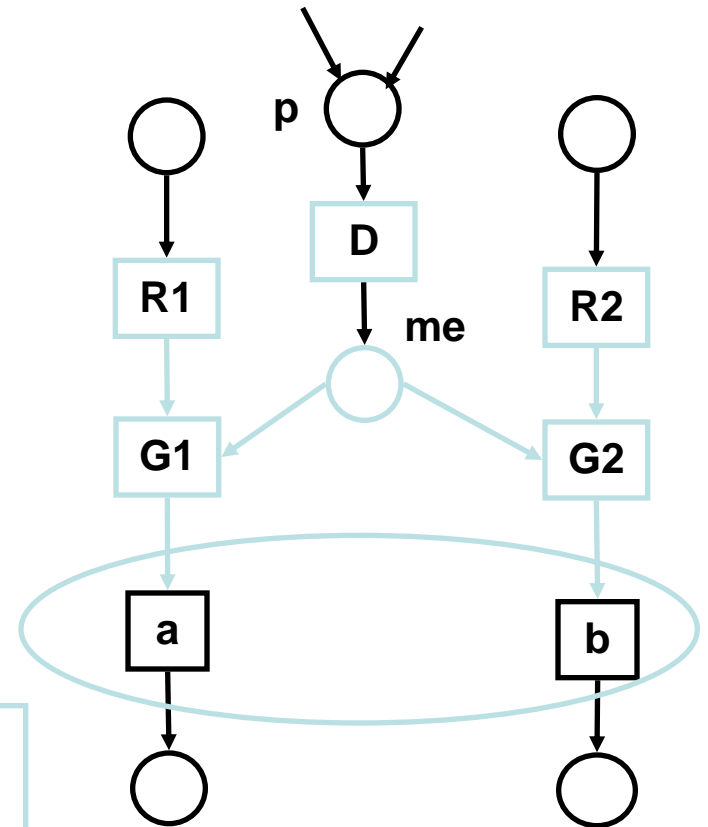
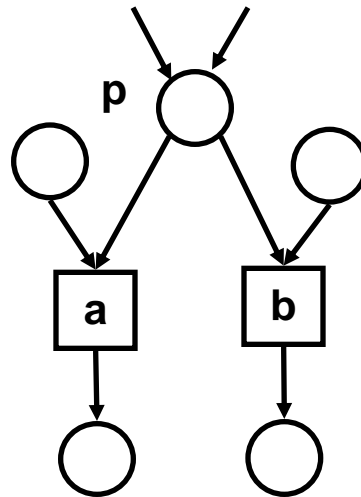
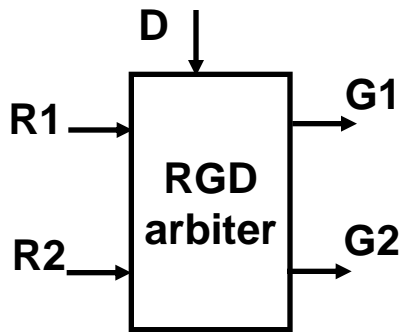


Assume **a** and **b** are circuit actions that are in conflict (may disable each other) and need to be protected



# Arbitration refinement

E.g. Request-Grant-Done (RGD) arbiter



**a and b are protected now (they are no longer disabled)**

# Translation of LPNs to circuits

- After appropriate refinements have been made one can translate Labelled Petri nets (or Signal Transition Graphs) into circuits
- Either by syntax-direct translation (discussed below)
- Or by using Logic Synthesis using STGs

# Why direct translation?

- Direct translation has linear complexity but can be area inefficient (inherent **one-hot** encoding)
- Logic synthesis has problems with state space explosion, repetitive and regular structures (**log**-based encoding approach)

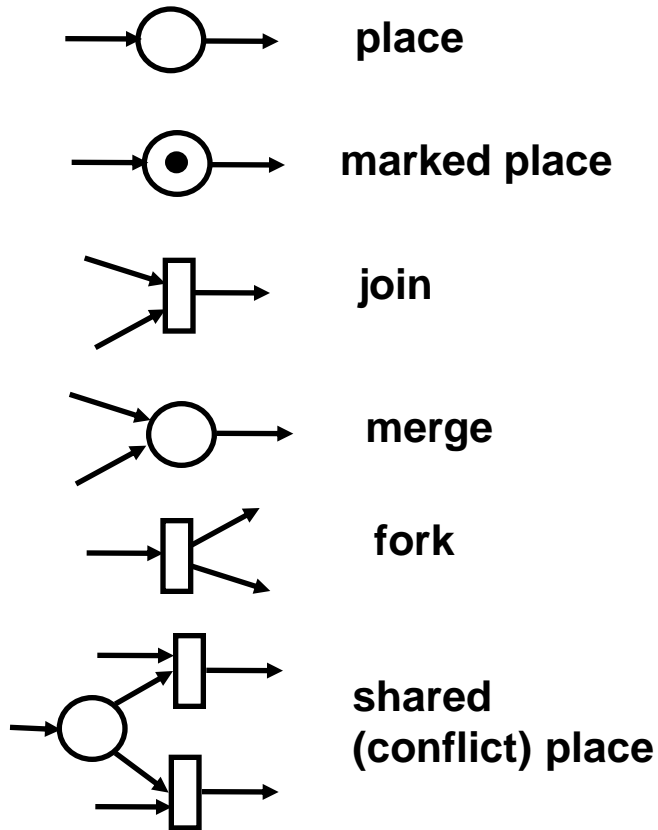


# Direct Translation of Petri Nets

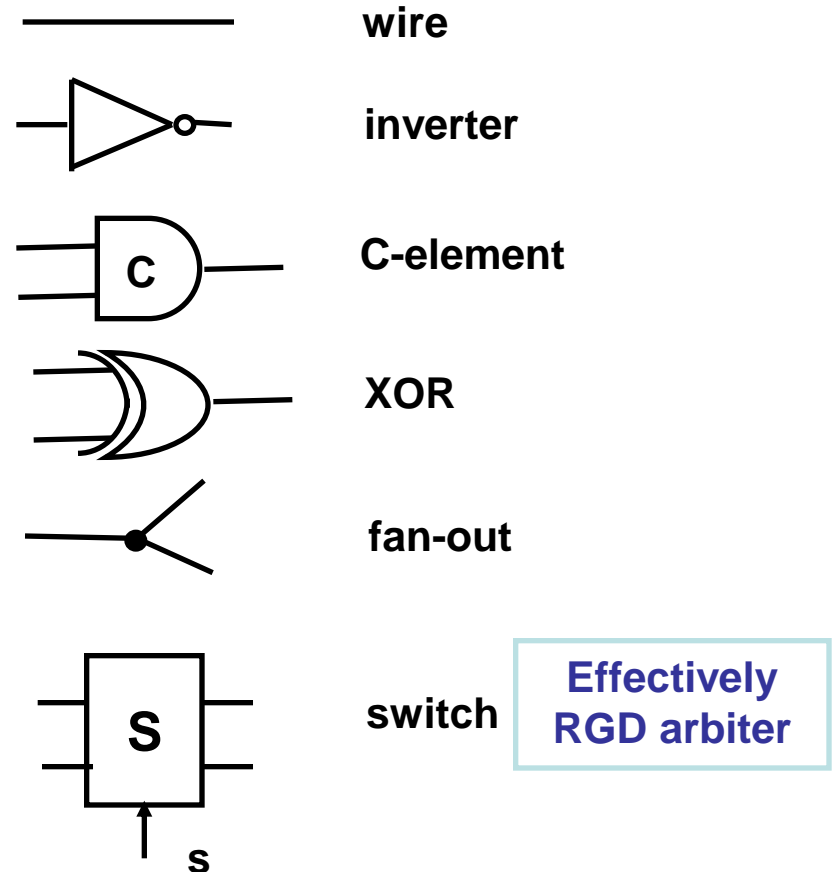
- Previous work dates back to 70s
- Synthesis into *event-based (two-phase)* circuits (similar to Sutherland's micropipeline control)
  - S.Patil, F.Furtek (MIT)
- Synthesis into *level-based (4-phase)* circuits (similar to synthesis from one-hot encoded FSMs)
  - R. David ('69, translation FSM graphs to CUSA cells)
  - L. Hollaar ('82, translation from parallel flowcharts)
  - V. Varshavsky et al. ('90,'96, translation from PN into an interconnection of David Cells)

# Patil's set of modules

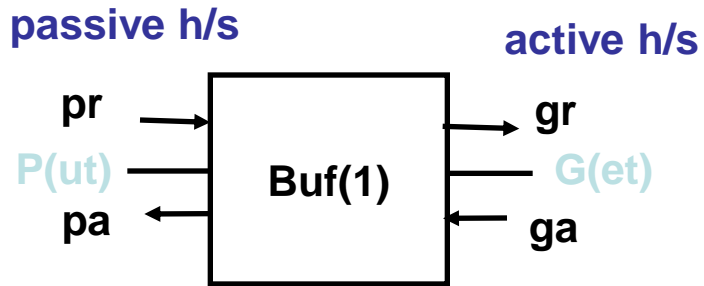
## Petri net fragment:



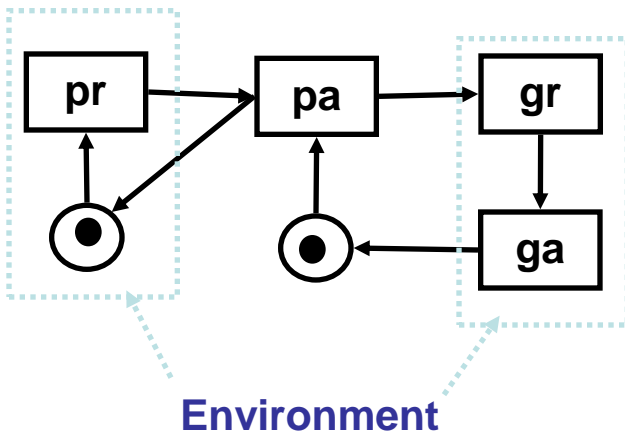
## Circuit equivalent:



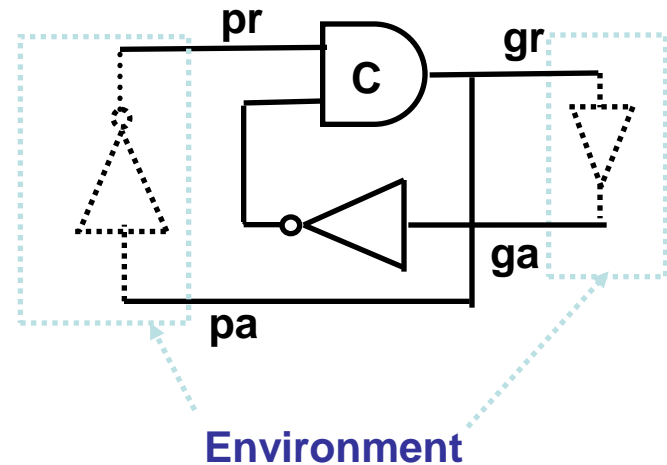
# Example



Two phase (NRZ) protocol:

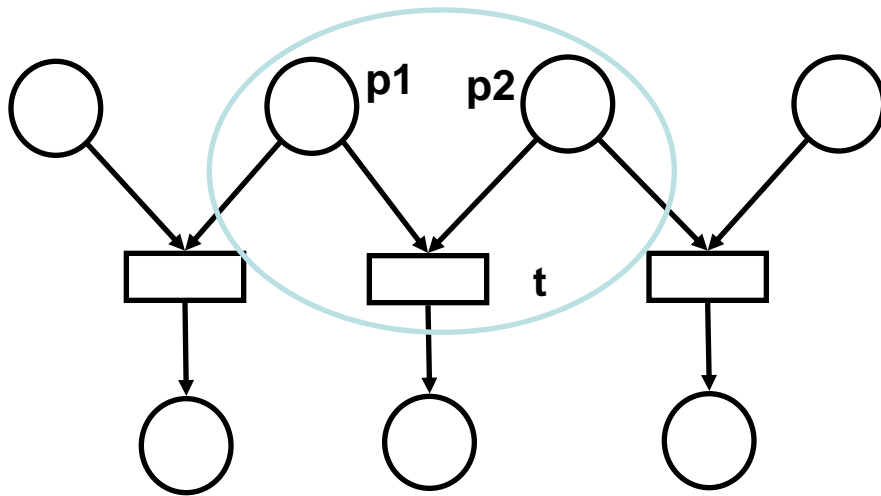


Two-phase implementation  
(using Patil's elements):



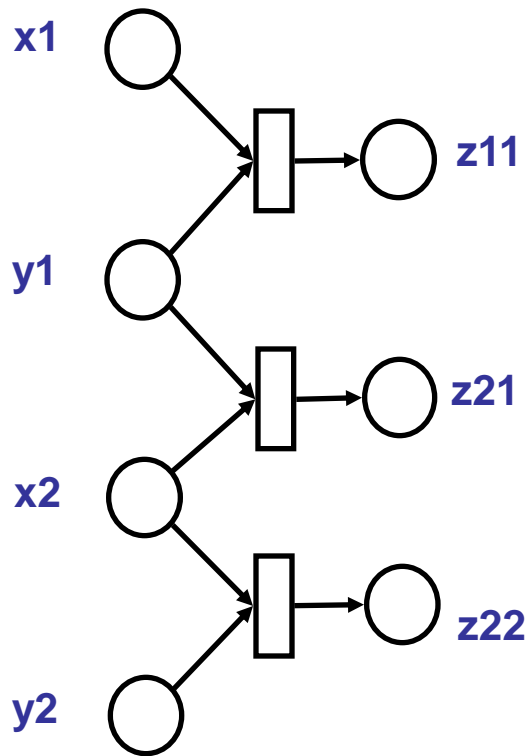
# Simple Net restriction

Patil's translation was restricted to (1-safe) Simple Nets

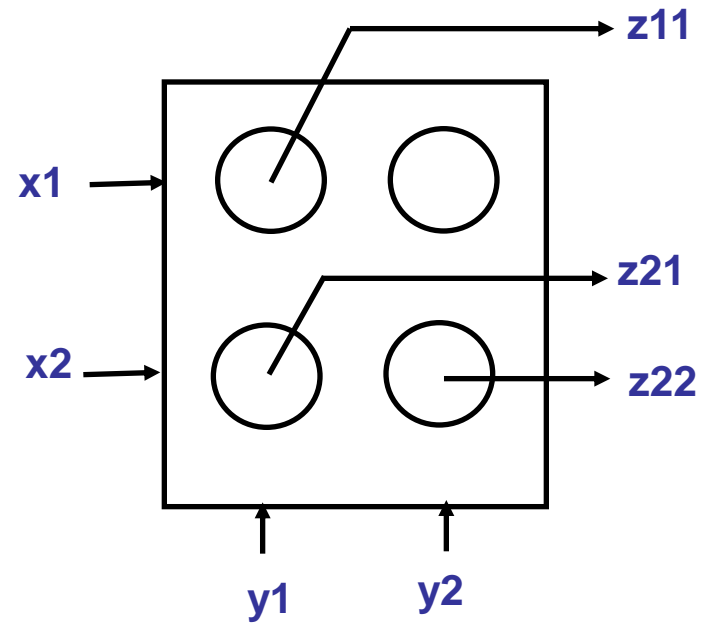


**Violation of simplicity: transition t has more than one input place (p1 and p2) that is input to other transitions**

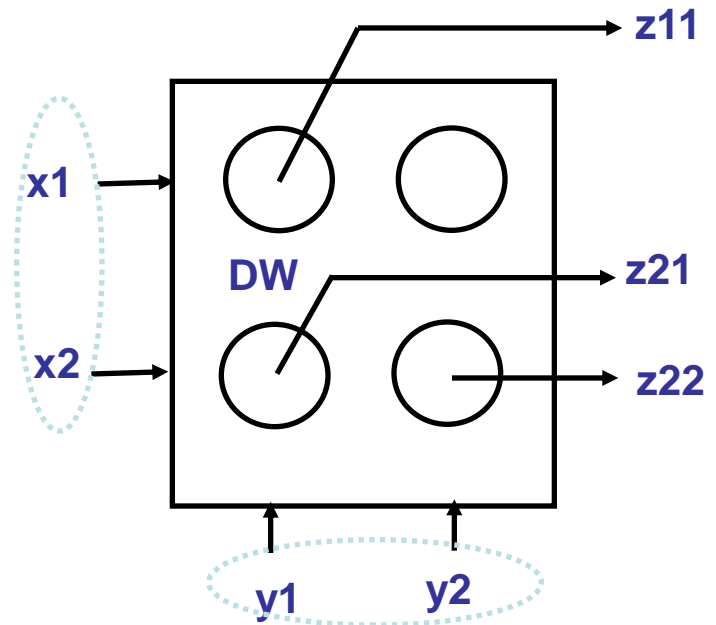
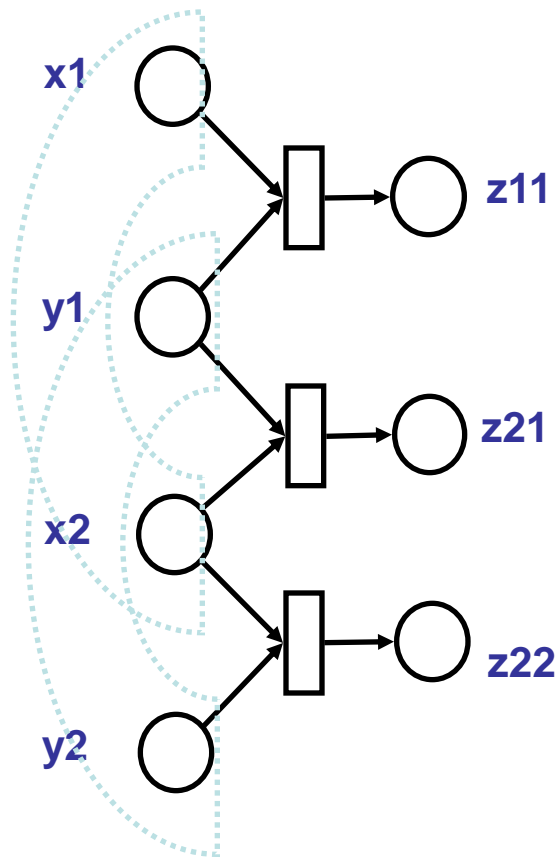
# Extension to Simple Nets



2-by-2 Decision-Wait element  
(multi-way Join)

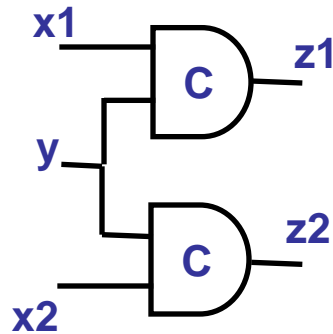
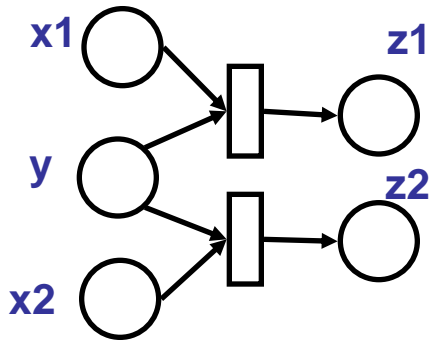


# Extension to Simple Nets



**$(x_1$  and  $x_2)$  and  $(y_1$  and  $y_2)$  must pairs of mutually exclusive events**

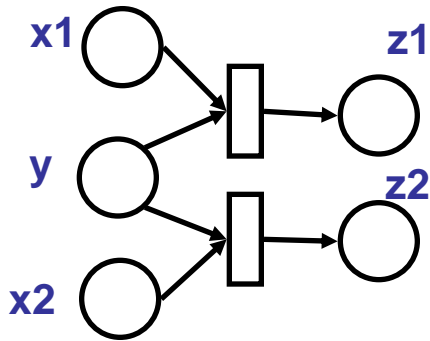
# Problems with C-elements



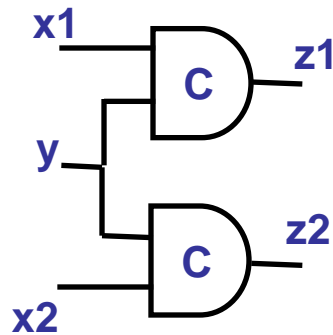
Can we just use a pair of C-elements to implement a 2-by-1 Decision wait?

$x1$  and  $x2$  are mutually exclusive – so no need for a S-switch (RGD arbiter)

# Problems with C-elements



$x1$  and  $x2$  are mutually exclusive – so no need for a S-switch (RGD arbiter)



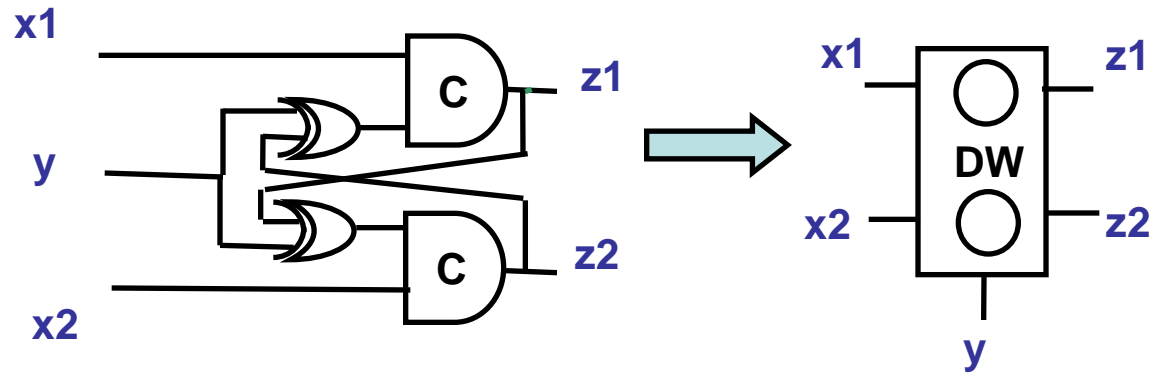
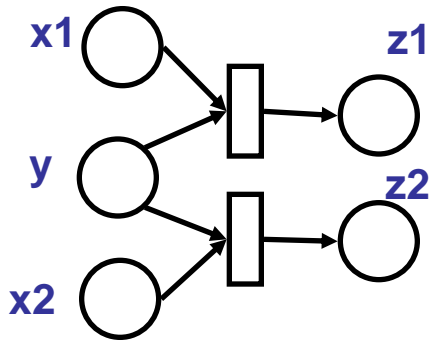
Can we just use a pair of C-elements to implement a 2-by-1 Decision wait?

No.

C-elements can only synchronise:  
rising (0-1) with rising (0-1) or  
falling (1-0) with falling (1-0) but  
not rising (0-1) with falling (1-0)



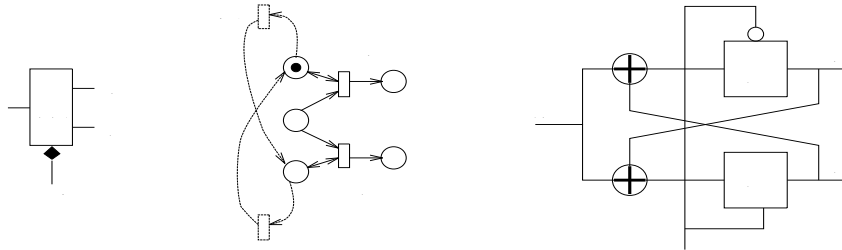
# Problems with C-elements



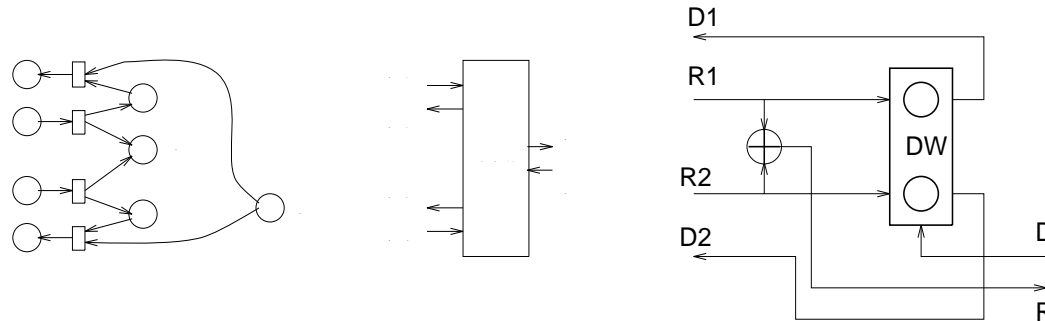
$x_1$  and  $x_2$  are mutually exclusive – so no need for a S-switch (RGD arbiter)

# Other useful elements

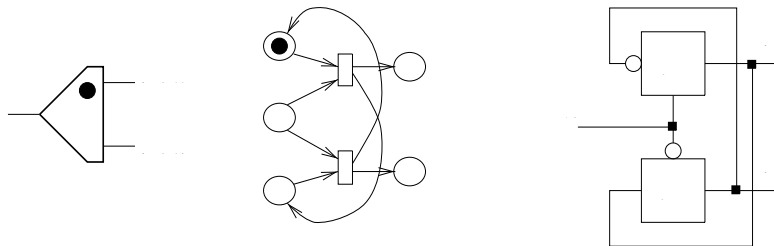
Select:



Call:

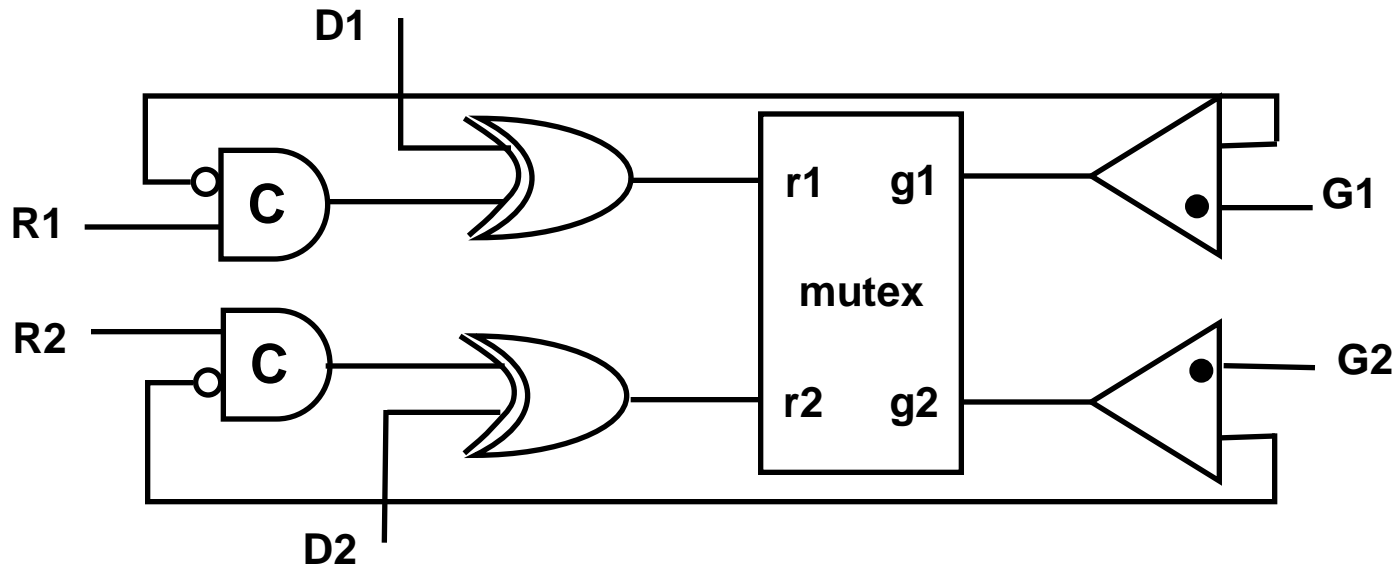


Toggle:

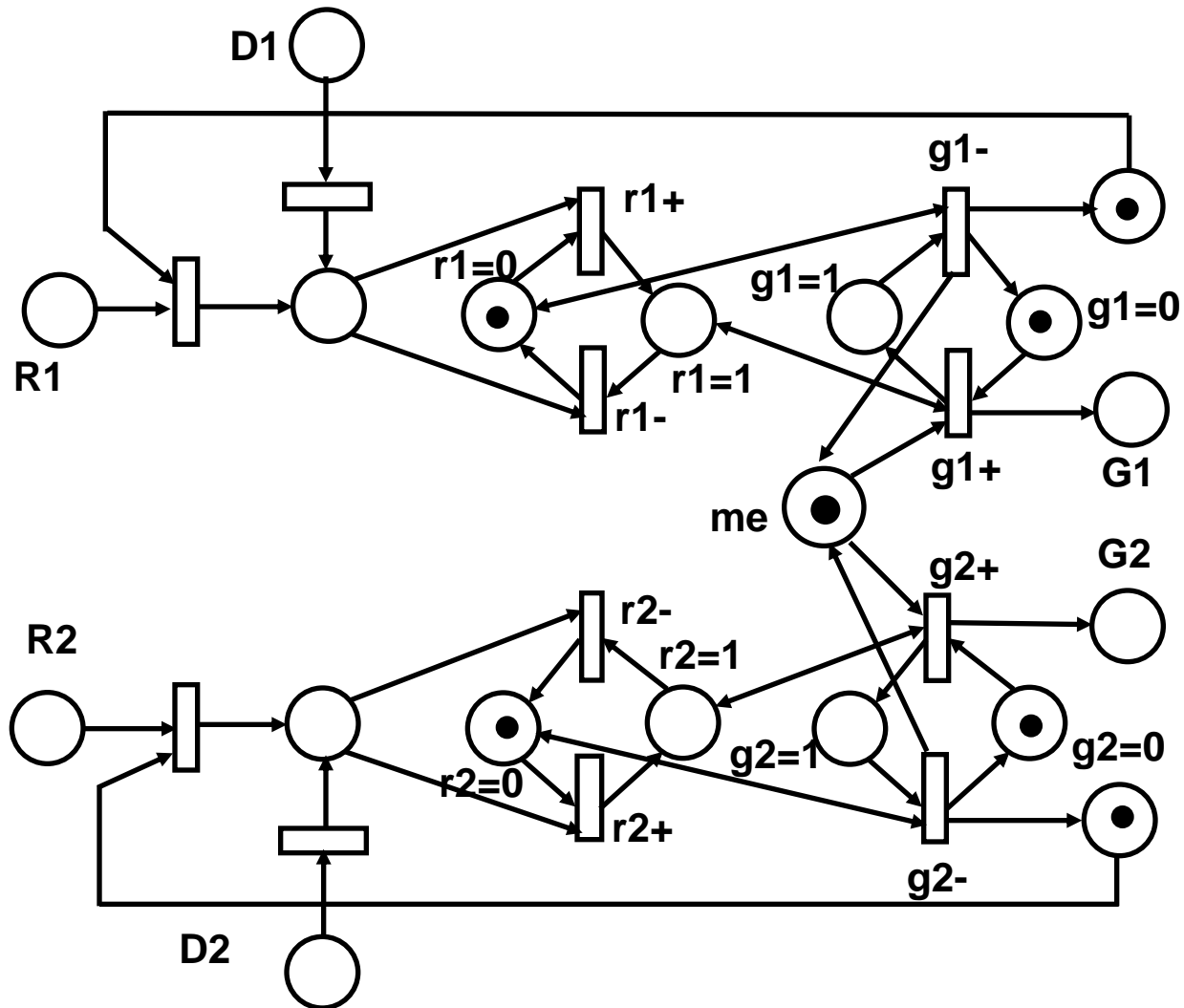


More examples

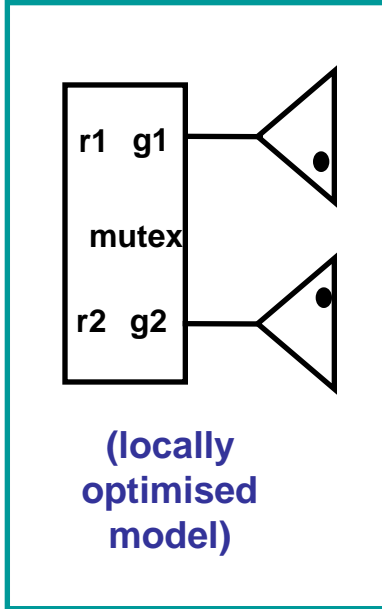
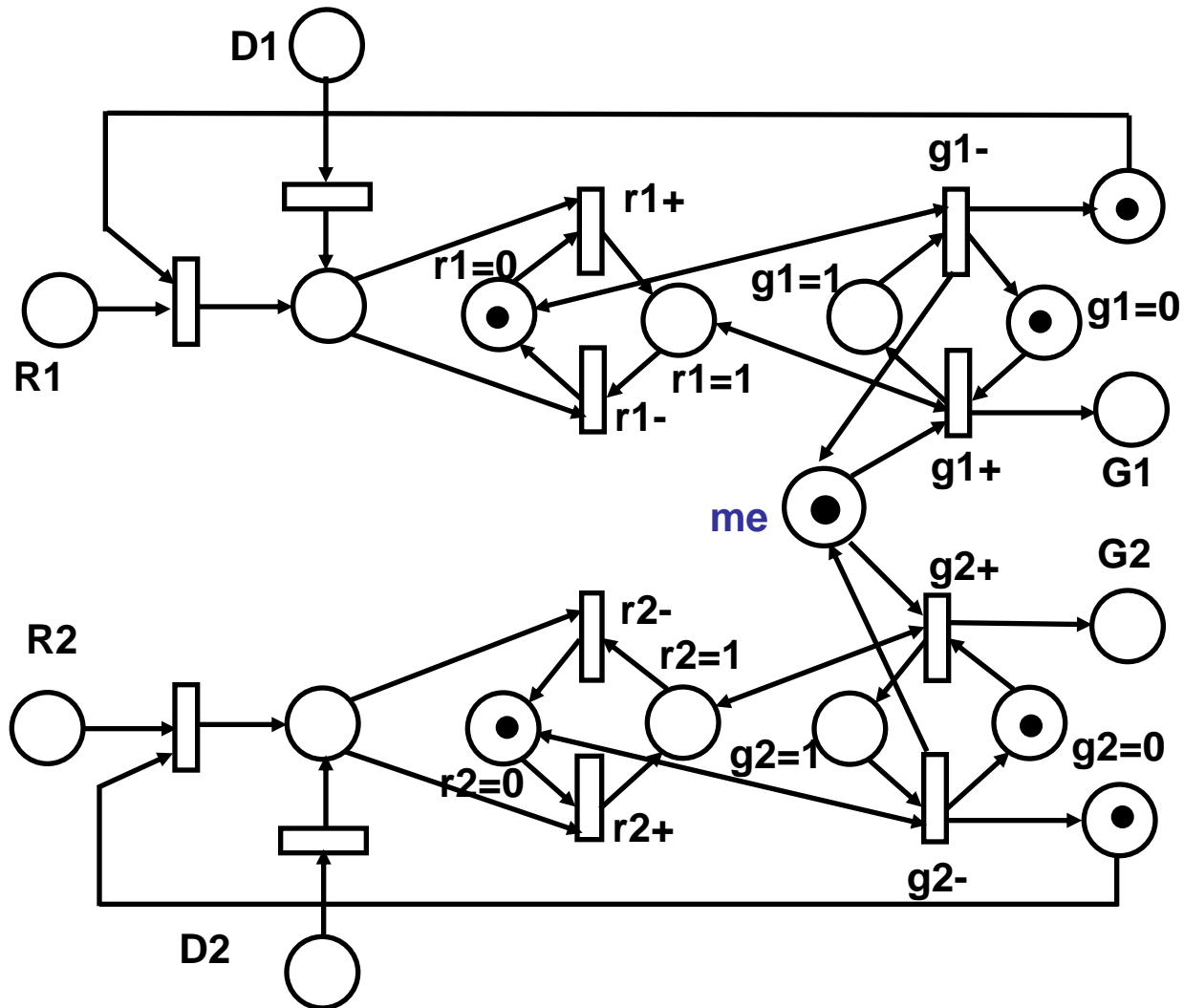
# Request-Grant-Done (RGD) arbiter



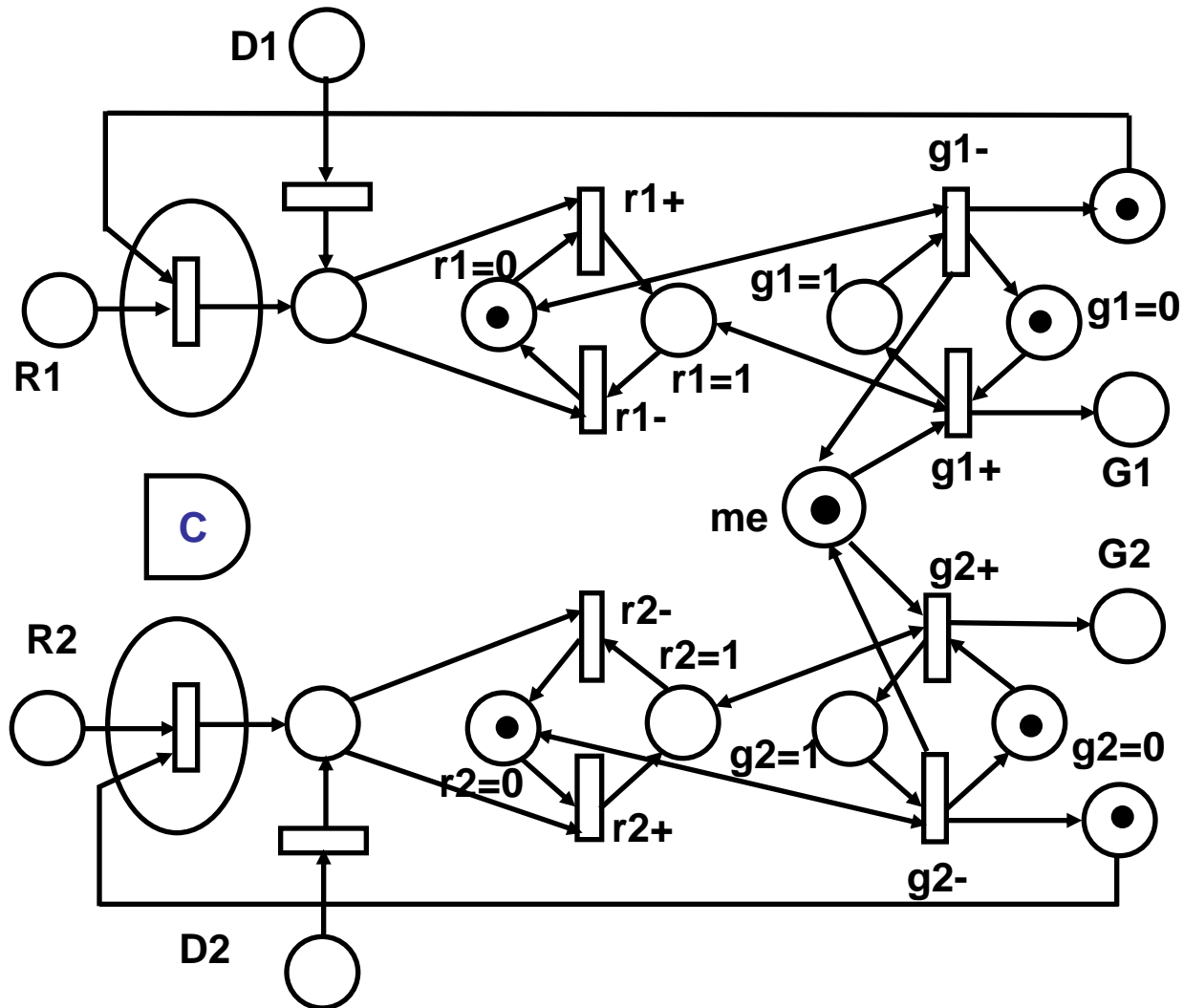
# Request-Grant-Done (RGD) arbiter



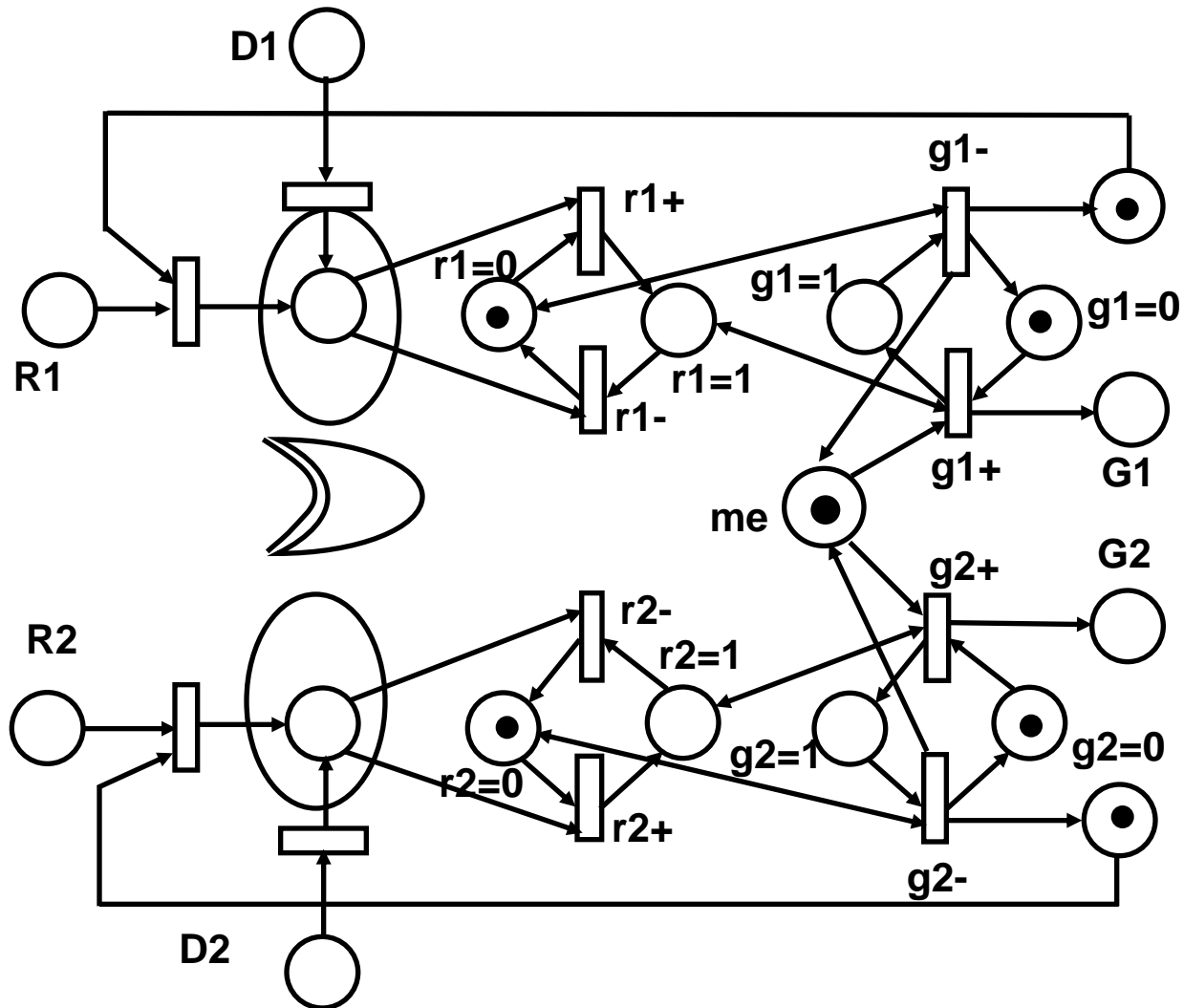
# Request-Grant-Done (RGD) arbiter



# Request-Grant-Done (RGD) arbiter

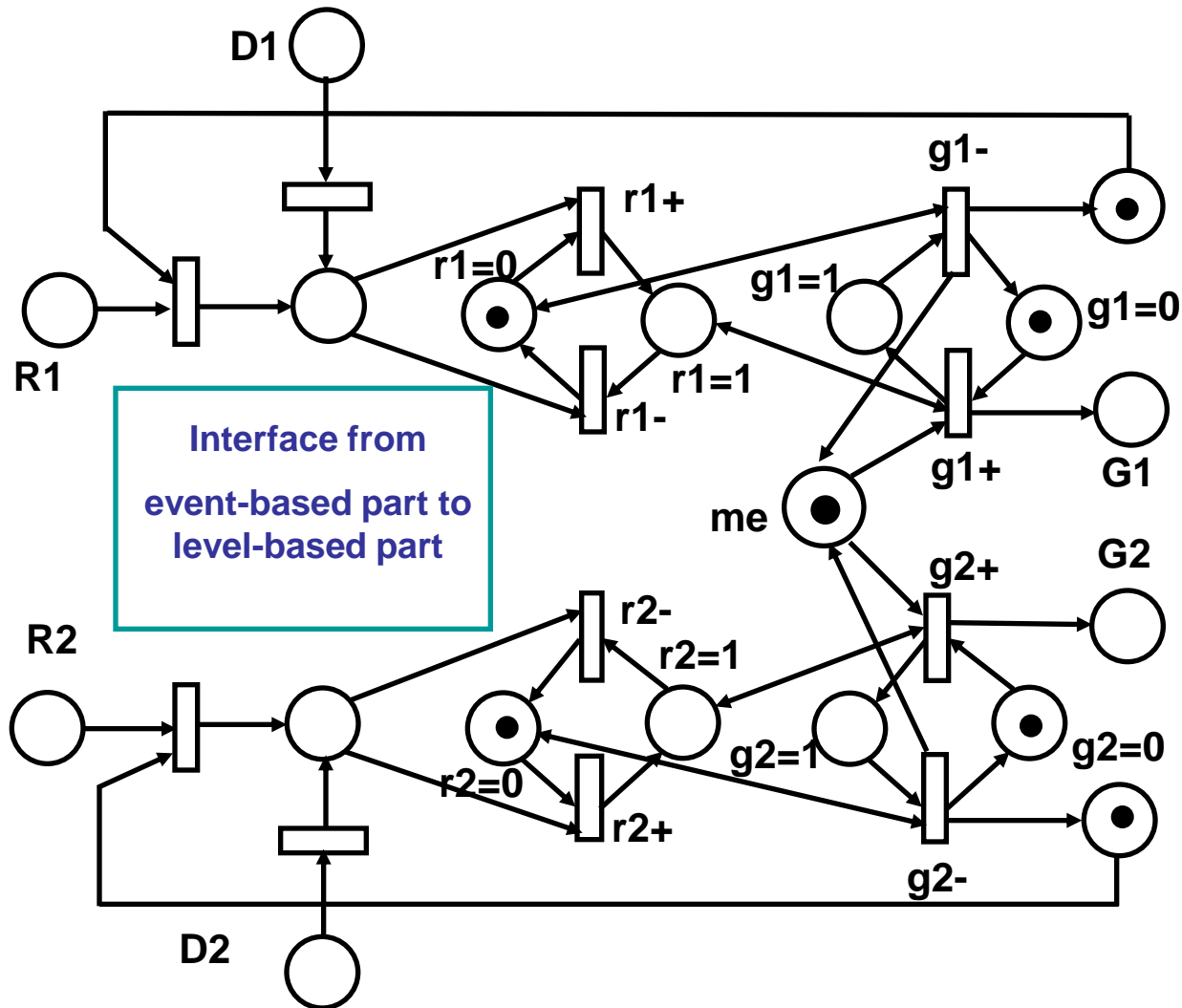


# Request-Grant-Done (RGD) arbiter

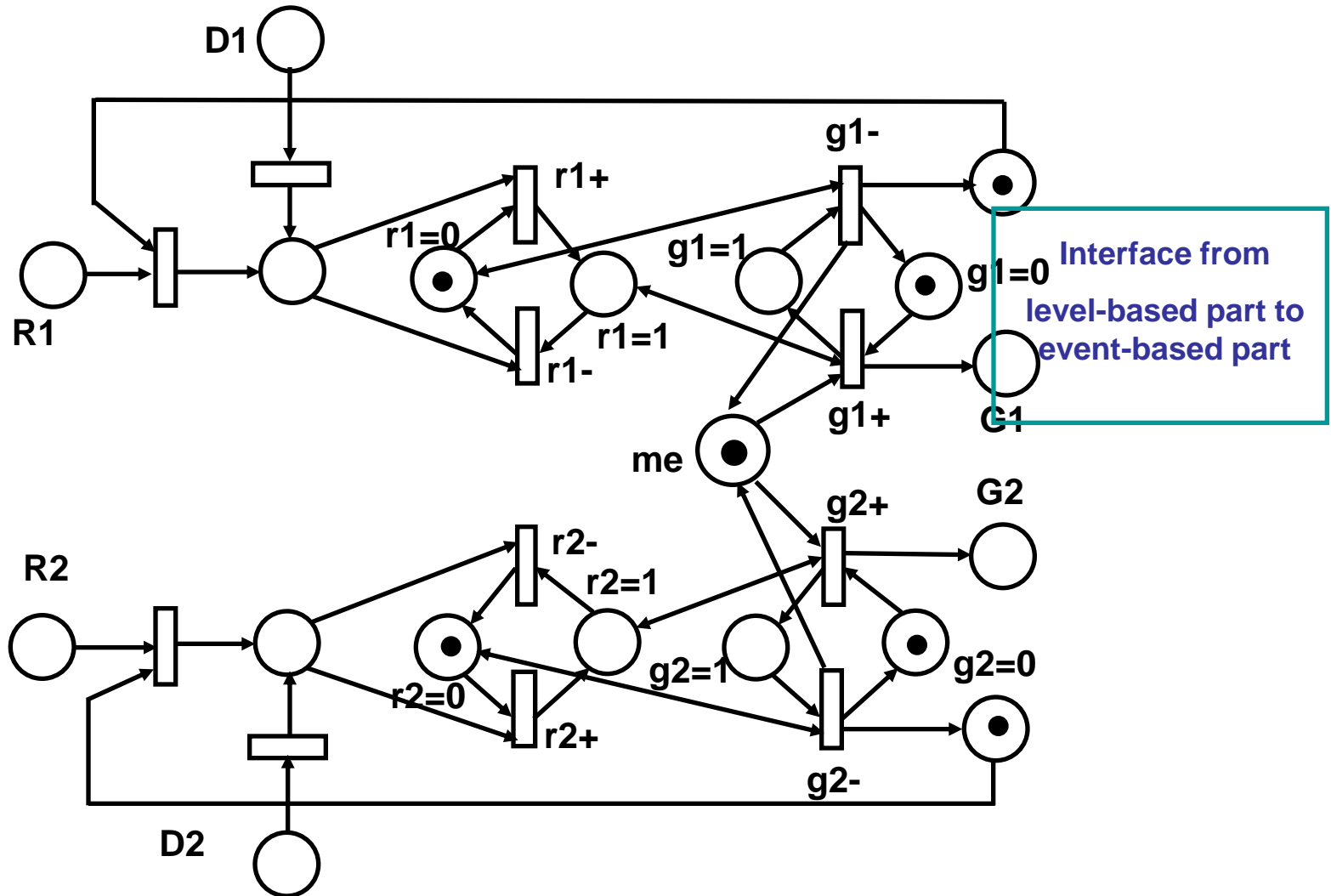




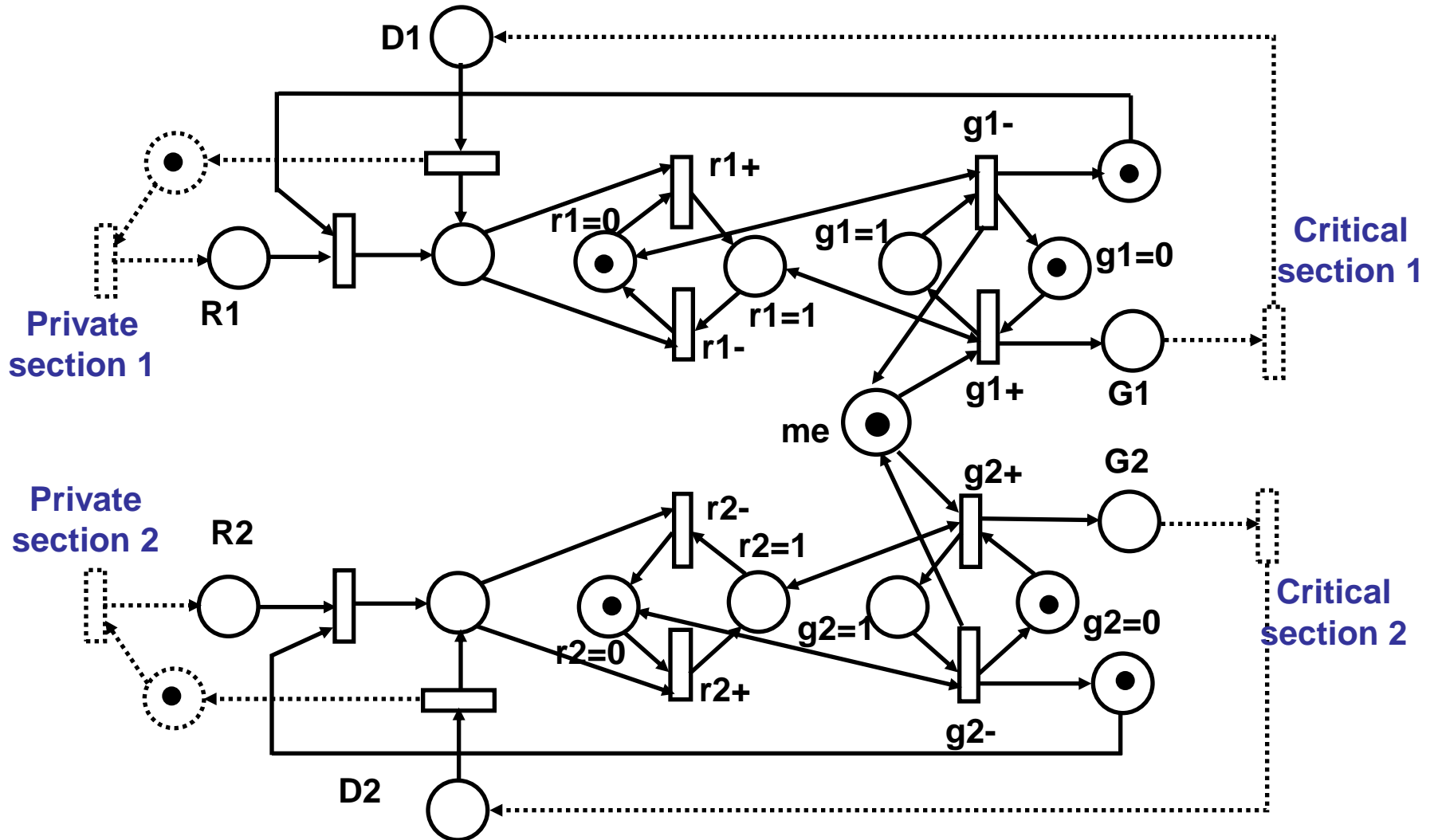
# Request-Grant-Done (RGD) arbiter



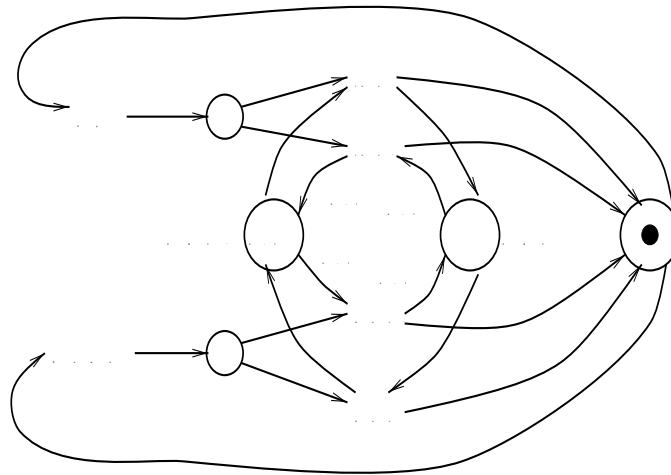
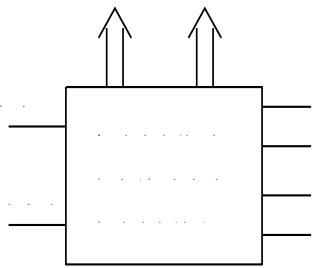
# Request-Grant-Done (RGD) arbiter



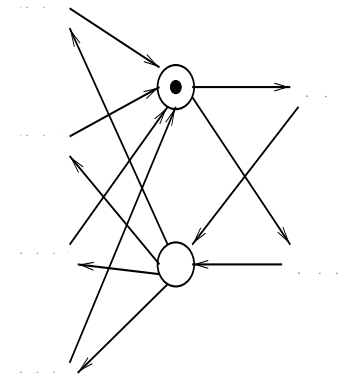
# Request-Grant-Done (RGD) arbiter with environment



# Direct synthesis example (modulo-k Up-Down counter)

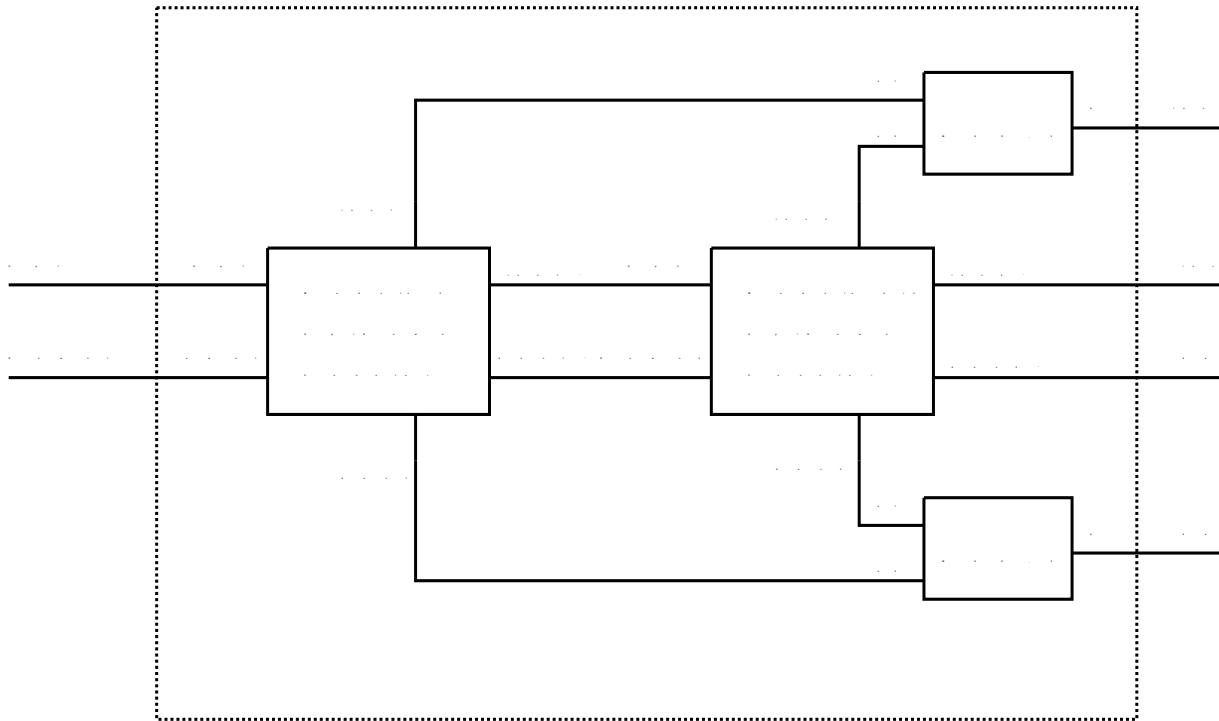


**Mod-k counter LPN**



**Environment LPN**

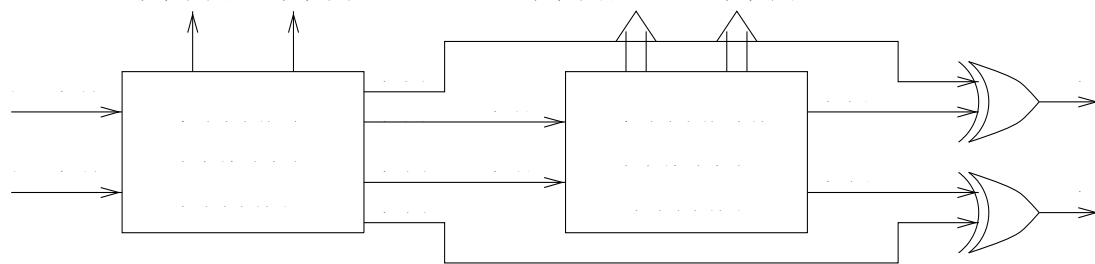
# Direct synthesis example (modulo-k Up-Down counter)



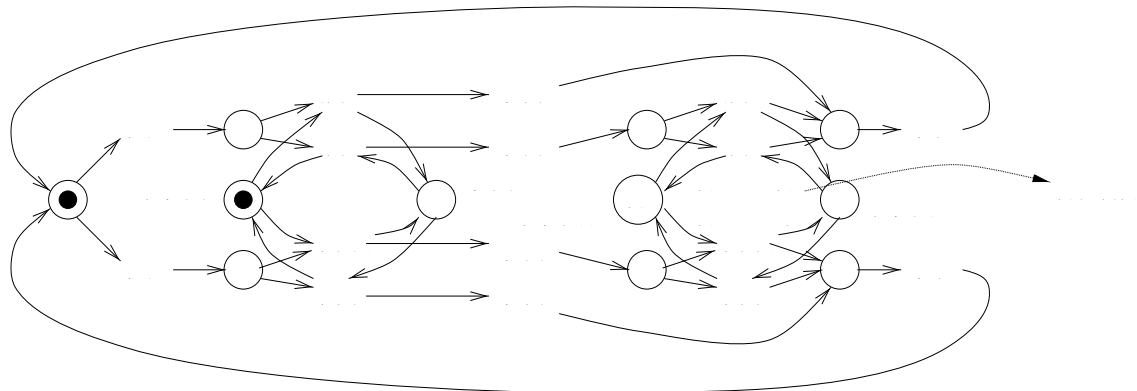
**Decomposition (structural view)**

# Direct synthesis example (modulo-k Up-Down counter)

structure

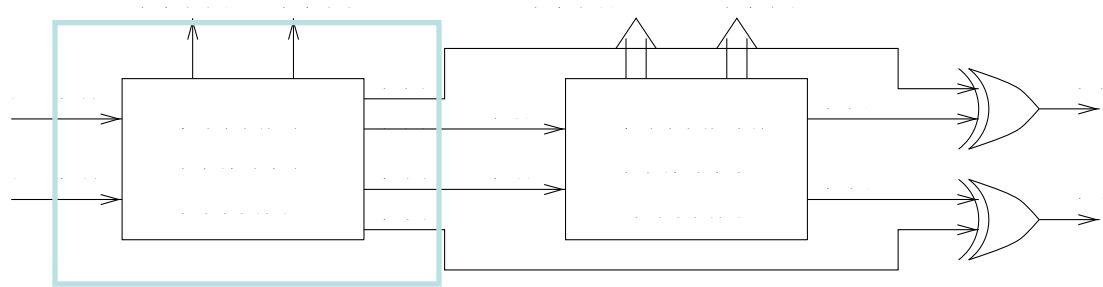


LPN

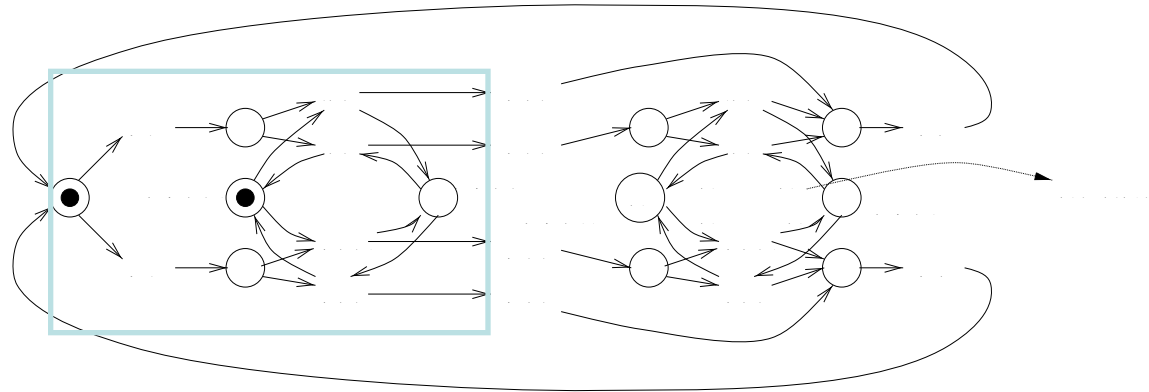


# Direct synthesis example (modulo-k Up-Down counter)

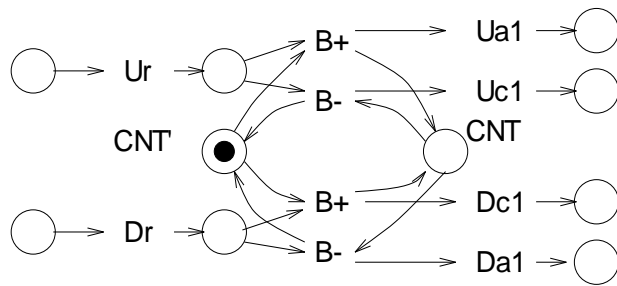
structure



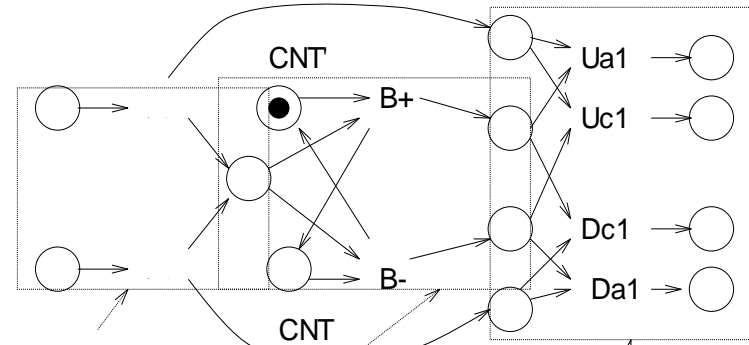
LPN



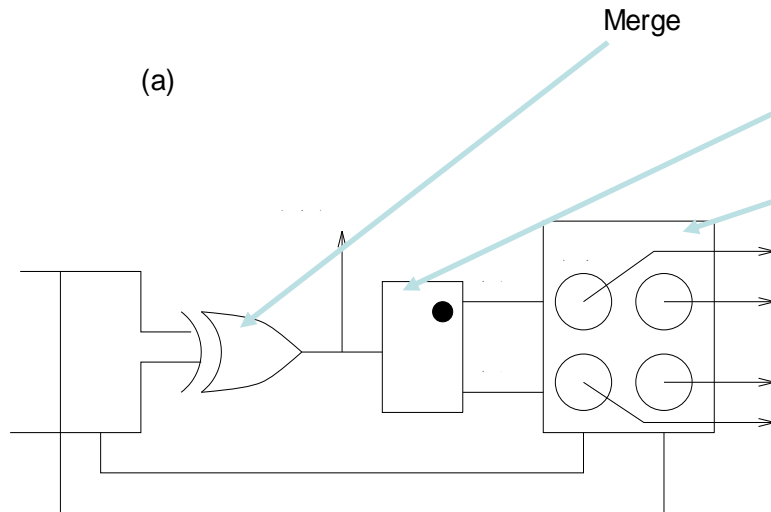
# Direct synthesis example (modulo-k Up-Down counter)



(a)



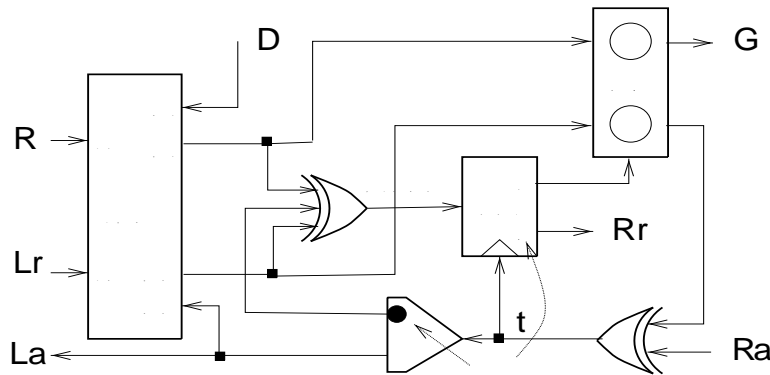
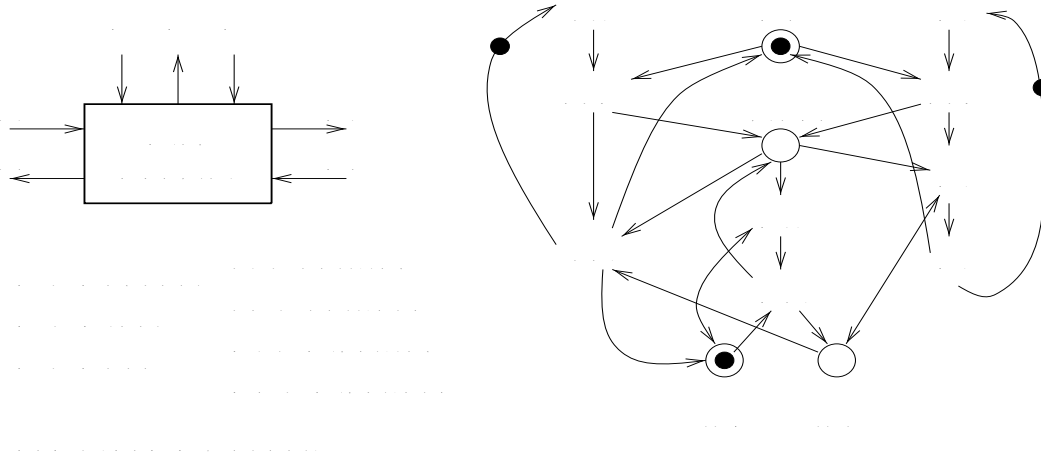
(b)



(c)

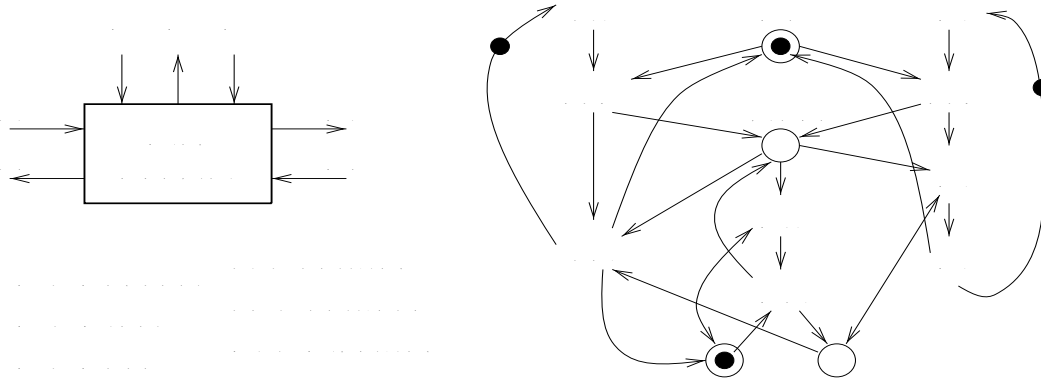


# Direct synthesis example (lazy token ring adapter)



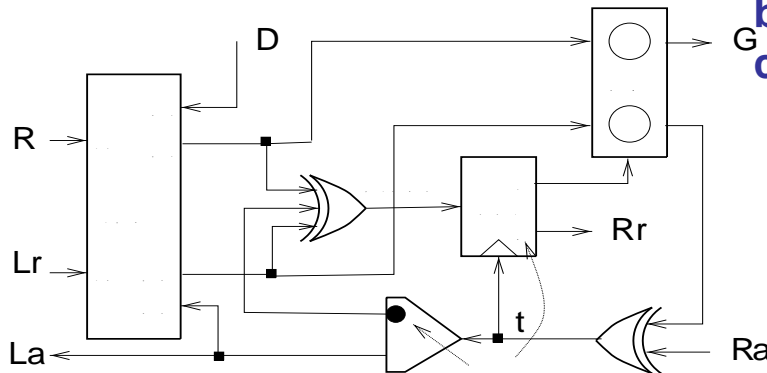
(b)

# Direct synthesis example (lazy token ring adapter)



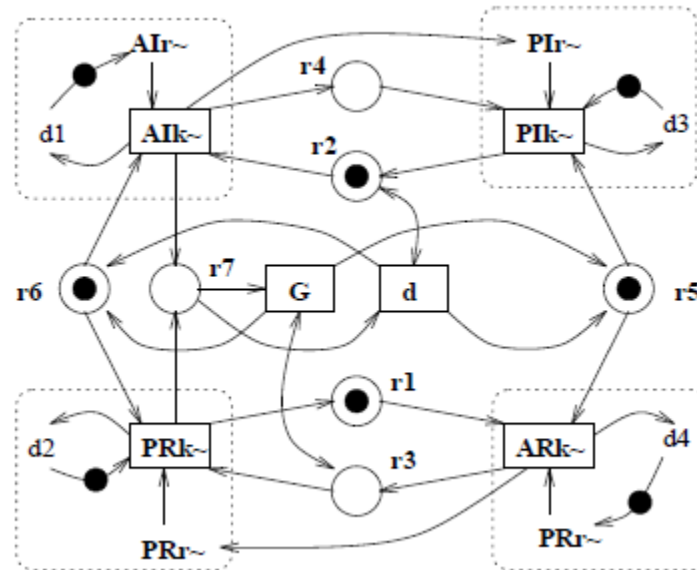
**Exercise:**

**Refine this initial LPN  
and map it (fragment-  
by-fragment) to the  
circuit**

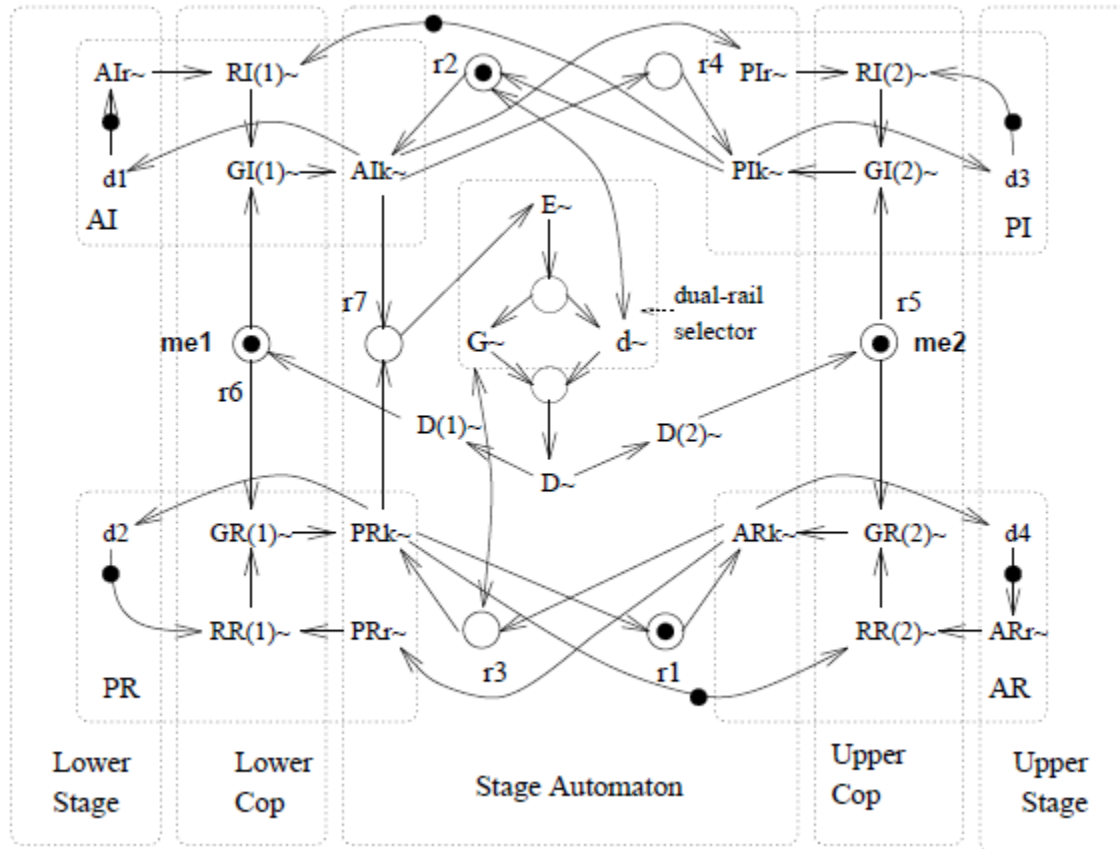


(b)

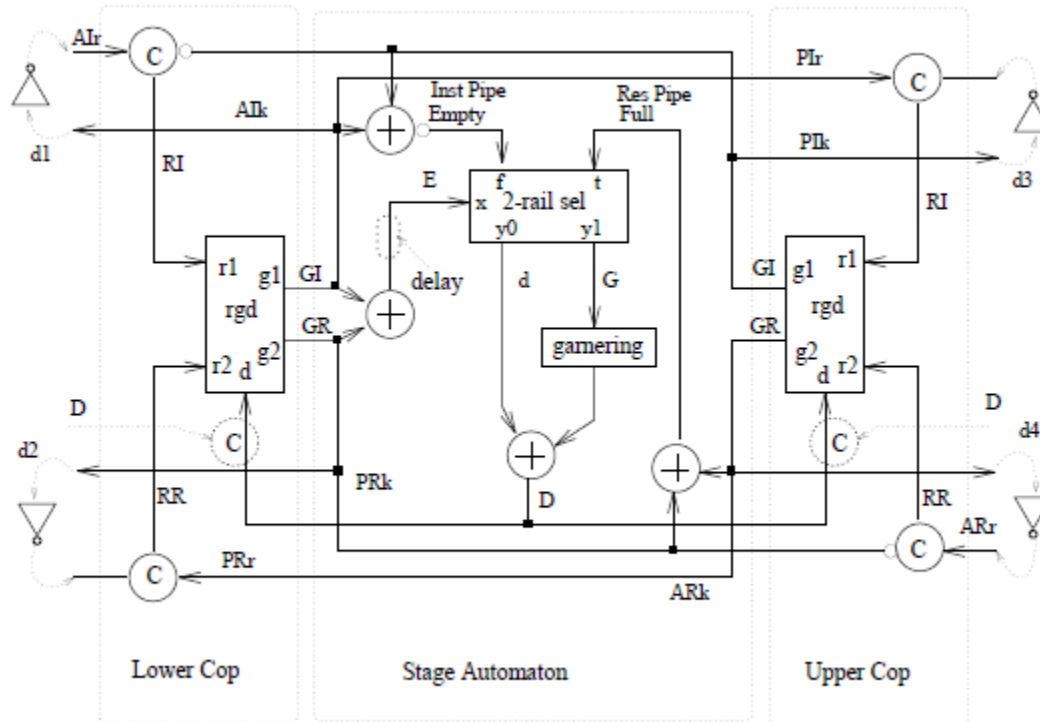
# Implementation of Counterflow Pipeline



# Counterflow pipeline implementation



# CFPP Implementation



# Workcraft

**GUI-based tool from Newcastle to work with interpreted graph models**

**Latest downloadable version -**

**<http://workcraft.org>**

# Tools at UPC, Barcelona

- **Petrify** (*J. Cortadella*)
  - <http://www.lsi.upc.edu/~jordicf/petrify/>
- **Genet** (*J. Carmona*)
  - <http://www.lsi.upc.edu/~jcarmona/genet.html>
- **Rbminer** (*M. Solé*)
  - <http://www.lsi.upc.edu/~jcarmona/rbminer/>

# Tools by other groups

- **SYNET** (*B. Caillaud*)
  - <http://www.irisa.fr/s4/tools/synet/>
- **VipTool** (*Bergenthum et al.*)
  - <http://viptool.ku-eichstaett.de/wiki/doku.php>
- **Parikh Miner** (*B. Van Dongen*)
  - <http://prom.win.tue.nl/research/wiki/prom/start>



# *Petrify* (J. Cortadella et al.)

- Features:
  - Synthesis/mining of safe Petri nets
  - Synthesis of asynchronous controllers
  - Stable, efficient, long-standing tool
  - Used in universities and industry.
- <http://www.isi.upc.edu/~jordicf/>
  - Binaries for Linux/Sun/Windows
  - Related papers
  - Tutorial.

# *Genet (J. Carmona)*

- Features:
  - Synthesis/mining of general Petri nets
  - Decompositional/Divide-and-Conquer approaches
  - GenetGUI: graphical user interface (by J. Muñoz)
- <http://www.lsi.upc.edu/~jcarmona/genet.html>
  - Binaries for Linux/Sun
  - Related papers
  - Tutorial.

# *Rbminer* (M. Solé)

- Features:
  - Mining of general Petri nets
  - Helper applications (log2ts, ...)
  - Efficiency
- <http://www.lsi.upc.edu/~jcarmona/rbminer/>
  - Binaries for Linux/
  - Related papers
  - Tutorial.